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Chapter 13

INDUCTION MACHINE TRANSIENTS

13.1. INTRODUCTION

Induction machines undergo transients when voltage, current, and (or) speed undergo changes. Turning on or off the power grid leads to starting transients in an induction motor.

Reconnecting an induction machine after a short-lived power fault (zero current) is yet another transient. Bus switching for large power induction machines feeding urgent loads also qualifies as large deviation transients.

Sudden short-circuits, at the terminals of large induction motors lead to very large peak currents and torques. On the other hand more and more induction motors are used in variable speed drives with fast electromagnetic and mechanical transients.

So, modeling transients is required for power-grid-fed (constant voltage and frequency) and for PWM converter-fed IM drives control.

Modeling the transients of induction machines may be carried out through circuit models or coupled field/circuit models (through FEM). We will deal first with phase-coordinate abc model with inductance matrix exhibiting terms dependent on rotor position.

Subsequently, the space phasor (d-q) model is derived. Both single and double rotor circuit models are dealt with. Saturation is also included in the space-phasor (d-q) model. The abc-dq model is then derived and applied, as it is adequate for nonsymmetrical voltage supplies and PWM converter-fed IMs.

Reduced order d-q models are used to simplify the study of transients for low and large motors, respectively.

Modeling transients with the computation of cage bar and end-ring currents is required when cage and/or end-ring faults occur. Finally the FEM coupled field circuit approach is dealt with.

Autonomous generator transients are left out as they are treated in the chapter dedicated to induction generators.

13.2. THE PHASE COORDINATE MODEL

The induction machine may be viewed as a system of electric and magnetic circuits which are coupled magnetically and/or electrically.

An assembly of resistances, self inductances, and mutual inductances is thus obtained. Let us first deal with the inductance matrix.

A symmetrical (healthy) cage may be replaced by a wound three-phase rotor. [2] Consequently, the IM is represented by six circuits, (phases) (Figure 14.1). Each of them is characterized by a self inductance and 5 mutual inductances.

The stator and rotor phase self inductances do not depend on rotor position if slot openings are neglected. Also, mutual inductances between stator phases and rotor phases, respectively, do not depend on rotor position. A sinusoidal distribution of windings is assumed. Finally, stator/rotor phase mutual inductances depend on rotor position ($\theta_{er} = p_1\theta_r$).

The induction matrix, $L_{abca_r b_r c_r}(\theta_{er})$ is

$$[L_{abca_r b_r c_r}(\theta_{er})] = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{aa_r} & L_{ab_r} & L_{ac_r} \\ L_{ab} & L_{bb} & L_{bc} & L_{ba_r} & L_{bb_r} & L_{bc_r} \\ L_{ac} & L_{bc} & L_{cc} & L_{ca_r} & L_{cb_r} & L_{cc_r} \\ L_{aa_r} & L_{ba_r} & L_{ca_r} & L_{a_r a_r} & L_{a_r b_r} & L_{a_r c_r} \\ L_{ab_r} & L_{bb_r} & L_{cb_r} & L_{a_r b_r} & L_{b_r b_r} & L_{b_r c_r} \\ L_{ac_r} & L_{bc_r} & L_{cc_r} & L_{a_r c_r} & L_{b_r c_r} & L_{c_r c_r} \end{bmatrix} \quad (13.1)$$

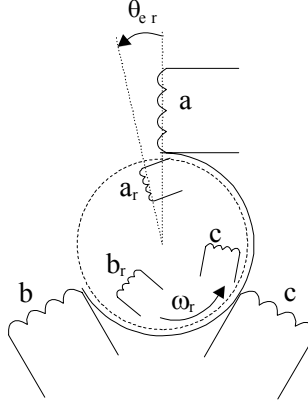


Figure 13.1 Three-phase IM with equivalent wound rotor

with

$$\begin{aligned} L_{aa} &= L_{bb} = L_{cc} = L_{ls} + L_{ms}; & L_{ab} &= L_{ac} = L_{bc} = -L_{ms}/2; \\ L_{aa_r} &= L_{bb_r} = L_{cc_r} = L_{srm} \cos \theta_{er}; & L_{a_r a_r} &= L_{b_r b_r} = L_{c_r c_r} = L_{lr}^r + L_{mr}^r; \\ L_{c_r a} &= L_{a_r b} = L_{b_r c} = L_{srm} \cos \left(\theta_{er} - \frac{2\pi}{3} \right); & L_{a_r b_r} &= L_{a_r c_r} = L_{b_r c_r} = -L_{mr}^r/2; \\ & & L_{c_r b} &= L_{b_r a} = L_{a_r c} = L_{srm} \cos \left(\theta_{er} + \frac{2\pi}{3} \right) \end{aligned} \quad (13.2)$$

Assuming a sinusoidal distribution of windings, it may be easily shown that

$$L_{srm} = \sqrt{L_{ms} \cdot L_{mr}^r} \quad (13.3)$$

Reducing the rotor to stator is useful especially for cage rotor IMs, no access to rotor variables is available.

In this case, the mutual inductance becomes equal to self inductance $L_{sm} \rightarrow L_{sm}$ and the rotor self inductance equal to the stator self inductance $L_{mr} \rightarrow L_{sm}$.

To conserve the fluxes and losses with stator reduced variables,

$$\frac{i_{ar}}{i_{ar}^r} = \frac{i_{br}}{i_{br}^r} = \frac{i_{cr}}{i_{cr}^r} = \frac{L_{sm}}{L_{sm}} = K_{rs} \quad (13.4)$$

$$\frac{V_{ar}}{V_{ar}^r} = \frac{V_{br}}{V_{br}^r} = \frac{V_{cr}}{V_{cr}^r} = \frac{i_{ar}^r}{i_{ar}} = \frac{i_{br}^r}{i_{br}} = \frac{i_{cr}^r}{i_{cr}} = \frac{1}{K_{rs}} \quad (13.5)$$

$$\frac{R_r}{R_r^r} = \frac{L_{lr}}{L_{lr}^r} = \frac{1}{K_{rs}^2} \quad (13.6)$$

The expressions of rotor resistance R_r , leakage inductance L_{lr} , both reduced to the stator for both cage and wound rotors are given in Chapter 6.

The same is true for R_s , L_{ls} of the stator. The magnetization self inductance L_{sm} has been calculated in Chapter 5.

Now the matrix form of phase coordinate (variable) model is

$$\begin{aligned} [V] &= [R][i] + \frac{d}{dt}[\Psi] \\ [V] &= [V_a, V_b, V_c, V_{ar}, V_{br}, V_{cr}]^T \\ [i] &= [i_a, i_b, i_c, i_{ar}, i_{br}, i_{cr}]^T \\ [R] &= \text{Diag}[R_s, R_s, R_s, R_r, R_r, R_r] \end{aligned} \quad (13.7)$$

$$[\Psi] = [L_{abca_r b_r c_r}(\theta_{er})][i] \quad (13.8)$$

$$[L_{abca_r b_r c_r}(\theta_{er})] = \begin{bmatrix} L_{ls} + L_{sm} & -L_{sm}/2 & -L_{sm}/2 & L_{sm} \cos \theta_{er} & L_{sm} \cos \left(\theta_{er} + \frac{2\pi}{3} \right) & L_{sm} \cos \left(\theta_{er} - \frac{2\pi}{3} \right) \\ -L_{sm}/2 & L_{ls} + L_{sm} & -L_{sm}/2 & L_{sm} \cos \left(\theta_{er} - \frac{2\pi}{3} \right) & L_{sm} \cos \theta_{er} & L_{sm} \cos \left(\theta_{er} + \frac{2\pi}{3} \right) \\ -L_{sm}/2 & -L_{sm}/2 & L_{ls} + L_{sm} & L_{sm} \cos \left(\theta_{er} + \frac{2\pi}{3} \right) & L_{sm} \cos \left(\theta_{er} - \frac{2\pi}{3} \right) & L_{sm} \cos \theta_{er} \\ L_{sm} \cos \theta_{er} & L_{sm} \cos \left(\theta_{er} - \frac{2\pi}{3} \right) & L_{sm} \cos \left(\theta_{er} + \frac{2\pi}{3} \right) & L_{ls} + L_{sm} & -L_{sm}/2 & -L_{sm}/2 \\ L_{sm} \cos \left(\theta_{er} + \frac{2\pi}{3} \right) & L_{sm} \cos \theta_{er} & L_{sm} \cos \left(\theta_{er} - \frac{2\pi}{3} \right) & -L_{sm}/2 & L_{ls} + L_{sm} & -L_{sm}/2 \\ L_{sm} \cos \left(\theta_{er} - \frac{2\pi}{3} \right) & L_{sm} \cos \left(\theta_{er} + \frac{2\pi}{3} \right) & L_{sm} \cos \theta_{er} & -L_{sm}/2 & -L_{sm}/2 & L_{ls} + L_{sm} \end{bmatrix} \quad (13.9)$$

With (13.8), (13.7) becomes

$$[V] = [R][i] + \left([L] + \left[\frac{\partial L}{\partial i} \right] [i] \right) \frac{d[i]}{dt} + \frac{d[L]}{d\theta_{er}} [i] \frac{d\theta_{er}}{dt} \quad (13.10)$$

Multiplying (13.10) by $[i]^T$ we get

$$[i]^T [V] = [i]^T R [i] + \frac{d}{dt} \left(\frac{1}{2} [L][i][i]^T \right) + \frac{1}{2} [i]^T \frac{d}{d\theta_{er}} [L][i] \omega_r \quad (13.11)$$

The first term represents the winding losses, the second, the stored magnetic energy variation, and the third, the electromagnetic power P_e .

$$P_e = T_e \frac{\omega_r}{p_1} = \frac{1}{2} [i]^T \frac{d[L]}{d\theta_{er}} [i] \omega_r \quad (13.12)$$

The electromagnetic torque T_e is

$$T_e = \frac{1}{2} p_1 [i]^T \frac{d[L]}{d\theta_{er}} [i] \quad (13.13)$$

The motion equation is

$$\frac{J}{p_1} \frac{d\omega_r}{dt} = T_e - T_{load}; \quad \frac{d\theta_{er}}{dt} = \omega_r \quad (13.14)$$

An 8th order nonlinear model with time-variable coefficients (inductances) has been obtained, even with core loss neglected.

Numerical methods are required to solve it, but the computation time is prohibitive. Consequently, the phase coordinate model is to be used only for special cases as the inductance and resistance matrix may be assigned any values and rotor position dependencies.

The complex or space variable model is now introduced to get rid of rotor position dependence of parameters.

13.3. THE COMPLEX VARIABLE MODEL

Let us use the following notations:

$$\begin{aligned} a &= e^{j\frac{2\pi}{3}}; \quad \cos \frac{2\pi}{3} = \text{Re}[a]; \quad \cos \frac{4\pi}{3} = \text{Re}[a^2] \\ \cos\left(\theta_{er} + \frac{2\pi}{3}\right) &= \text{Re}[ae^{j\theta_{er}}] \quad \cos\left(\theta_{er} + \frac{4\pi}{3}\right) = \text{Re}[a^2 e^{j\theta_{er}}] \end{aligned} \quad (13.15)$$

Based on the inductance matrix, expression (13.9), the stator phase a and rotor phase a_r flux linkages Ψ_a and Ψ_{ar} are

$$\Psi_a = L_{ls} i_a + L_{ms} \text{Re}[i_a + ai_b + a^2 i_c] + L_{ms} \text{Re}[(i_{ar} + ai_{br} + a^2 i_{cr}) e^{j\theta_{er}}] \quad (13.16)$$

$$\Psi_{a_r} = L_{lr} \dot{i}_{a_r} + L_{ms} \operatorname{Re}[\dot{i}_{a_r} + a \dot{i}_{b_r} + a^2 \dot{i}_{c_r}] + L_{ms} \operatorname{Re}[(\dot{i}_a + a \dot{i}_b + a^2 \dot{i}_c) e^{-j\theta_{er}}] \quad (13.17)$$

We may now introduce the following complex variables as space phasors:
[1]

$$\vec{i}_s^s = \frac{2}{3} (\dot{i}_a + a \dot{i}_b + a^2 \dot{i}_c) \quad (13.18)$$

$$\vec{i}_r^r = \frac{2}{3} (\dot{i}_{a_r} + a \dot{i}_{b_r} + a^2 \dot{i}_{c_r}) \quad (13.19)$$

Also,

$$\operatorname{Re}(\vec{i}_s^s) = \dot{i}_a - \frac{1}{3} (\dot{i}_a + \dot{i}_b + \dot{i}_c) \quad (13.20)$$

$$\operatorname{Re}(\vec{i}_r^r) = \dot{i}_{a_r} - \frac{1}{3} (\dot{i}_{a_r} + \dot{i}_{b_r} + \dot{i}_{c_r}) \quad (13.21)$$

In symmetric steady-state and transient regimes,

$$\dot{i}_a + \dot{i}_b + \dot{i}_c = \dot{i}_{a_r} + \dot{i}_{b_r} + \dot{i}_{c_r} = 0 \quad (13.22)$$

With the above definitions, Ψ_a and Ψ_{ar} become

$$\Psi_a = L_{ls} \operatorname{Re}(\vec{i}_s^s) + L_m \operatorname{Re}(\vec{i}_s^s + \vec{i}_r^r e^{j\theta_{er}}); \quad L_m = \frac{3}{2} L_{ms} \quad (13.23)$$

$$\Psi_{ar} = L_{lr} \operatorname{Re}(\vec{i}_r^r) + L_m \operatorname{Re}(\vec{i}_r^r + \vec{i}_s^s e^{-j\theta_{er}}) \quad (13.24)$$

Similar expressions may be derived for phases b_r and c_r . After adding them together, using the complex variable definitions (13.18) and (13.19) for flux linkages and voltages, also, we obtain

$$\begin{aligned} \overline{V}_s^s &= R_s \vec{i}_s^s + \frac{d\overline{\Psi}_s^s}{dt}; \quad \overline{\Psi}_s^s = L_s \vec{i}_s^s + L_m \vec{i}_r^r e^{j\theta_{er}}; \\ \overline{V}_r^r &= R_r \vec{i}_r^r + \frac{d\overline{\Psi}_r^r}{dt}; \quad \overline{\Psi}_r^r = L_r \vec{i}_r^r + L_m \vec{i}_s^s e^{-j\theta_{er}} \end{aligned} \quad (13.25)$$

where

$$L_s = L_{sl} + L_m; \quad L_r = L_{rl} + L_m \quad (13.26)$$

$$\overline{V}_s^s = \frac{2}{3} (V_a + a V_b + a^2 V_c); \quad \overline{V}_r^r = \frac{2}{3} (V_{a_r} + a V_{b_r} + a^2 V_{c_r}) \quad (13.27)$$

In the above equations, stator variables are still given in stator coordinates and rotor variables in rotor coordinates.

Making use of a rotation of complex variables by the general angle θ_b in the stator and $\theta_b - \theta_{er}$ in the rotor, we obtain all variables in a unique reference rotating at electrical speed ω_b ,

$$\omega_b = \frac{d\theta_b}{dt} \quad (13.28)$$

$$\begin{aligned} \bar{\Psi}_s^s &= \bar{\Psi}_s^b e^{j\theta_b}; & \bar{i}_s^s &= \bar{i}_s^b e^{j\theta_b}; & \bar{V}_s^s &= \bar{V}_s^b e^{j\theta_b}; \\ \bar{\Psi}_r^r &= \bar{\Psi}_r^b e^{j(\theta_b - \theta_{er})}; & \bar{i}_r^r &= \bar{i}_r^b e^{j(\theta_b - \theta_{er})}; & \bar{V}_r^r &= \bar{V}_r^b e^{j(\theta_b - \theta_{er})} \end{aligned} \quad (13.29)$$

With these new variables Equations (13.25) become

$$\begin{aligned} \bar{V}_s &= R_s \bar{i}_s + \frac{d\bar{\Psi}_s}{dt} + j\omega_b \bar{\Psi}_s; & \bar{\Psi}_s &= L_s \bar{i}_s + L_m \bar{i}_r \\ \bar{V}_r &= R_r \bar{i}_r + \frac{d\bar{\Psi}_r}{dt} + j(\omega_b - \omega_r) \bar{\Psi}_r; & \bar{\Psi}_r &= L_r \bar{i}_r + L_m \bar{i}_s \end{aligned} \quad (13.30)$$

For convenience, the superscript b was dropped in (13.30). The electromagnetic torque is related to motion-induced voltage in (13.30).

$$T_e = \frac{3}{2} \cdot p_l \cdot \text{Re}(j \cdot \bar{\Psi}_s \cdot \bar{i}_s^*) = -\frac{3}{2} \cdot p_l \cdot \text{Re}(j \cdot \bar{\Psi}_r \cdot \bar{i}_r^*) \quad (13.31)$$

Adding the equations of motion, the complete complex variable (space-phasor) model of IM is obtained.

$$\frac{J}{p_l} \frac{d\omega_r}{dt} = T_e - T_{\text{load}}; \quad \frac{d\theta_{er}}{dt} = \omega_r \quad (13.32)$$

The complex variables may be decomposed in plane along two orthogonal d and q axes rotating at speed ω_b to obtain the d-q (Park) model. [2]

$$\begin{aligned} \bar{V}_s &= V_d + j \cdot V_q; & \bar{i}_s &= i_d + j \cdot i_q; & \bar{\Psi}_s &= \Psi_d + j \cdot \Psi_q \\ \bar{V}_r &= V_{dr} + j \cdot V_{qr}; & \bar{i}_r &= i_{dr} + j \cdot i_{qr}; & \bar{\Psi}_r &= \Psi_{dr} + j \cdot \Psi_{qr} \end{aligned} \quad (13.33)$$

With (13.33), the voltage Equations (13.30) become

$$\begin{aligned}
\frac{d\Psi_d}{dt} &= V_d - R_s \cdot i_d + \omega_b \cdot \Psi_q \\
\frac{d\Psi_q}{dt} &= V_q - R_s \cdot i_q - \omega_b \cdot \Psi_d \\
\frac{d\Psi_{dr}}{dt} &= V_{dr} - R_r \cdot i_{dr} + (\omega_b - \omega_r) \cdot \Psi_{qr} \\
\frac{d\Psi_{qr}}{dt} &= V_{qr} - R_r \cdot i_{qr} - (\omega_b - \omega_r) \cdot \Psi_{dr} \\
T_e &= \frac{3}{2} p_l (\Psi_d i_q - \Psi_q i_d) = \frac{3}{2} p_l L_m (i_q i_{dr} - i_d i_{qr})
\end{aligned} \tag{13.34}$$

Also from (13.27) with (13.19), the Park transformation for stator $P(\theta_b)$ is derived.

$$\begin{bmatrix} V_d \\ V_q \\ V_0 \end{bmatrix} = [P(\theta_b)] \cdot \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \tag{13.35}$$

$$[P(\theta_b)] = \frac{2}{3} \cdot \begin{bmatrix} \cos(-\theta_b) & \cos\left(-\theta_b + \frac{2\pi}{3}\right) & \cos\left(-\theta_b - \frac{2\pi}{3}\right) \\ \sin(-\theta_b) & \sin\left(-\theta_b + \frac{2\pi}{3}\right) & \sin\left(-\theta_b - \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{13.36}$$

The inverse Park transformation is

$$[P(\theta_b)]^{-1} = \frac{3}{2} \cdot [P(\theta_b)]^T \tag{13.37}$$

A similar transformation is valid for the rotor but with $\theta_b - \theta_{er}$ instead of θ_b .

It may be easily proved that the homopolar (real) variables $V_0, i_0, V_{0r}, i_{0r}, \Psi_0, \Psi_{0r}$ do not interface in energy conversion

$$\begin{aligned}
\frac{d\Psi_0}{dt} &= V_0 - R_s \cdot i_0; \quad \Psi_0 \approx L_{0s} \cdot i_0 \\
\frac{d\Psi_{0r}}{dt} &= V_{0r} - R_r \cdot i_{0r}; \quad \Psi_{0r} \approx L_{0r} \cdot i_{0r}
\end{aligned} \tag{13.38}$$

L_{0s} and L_{0r} are the homopolar inductances of stator and rotor. Their values are equal or lower (for chorded coil windings) to the respective leakage inductances L_{ls} and L_{lr} .

A few remarks on the complex variable (space phasor) and d-q models are in order.

- Both models include, in the form presented here, only the space fundamental of mmfs and airgap flux distributions.
- Both models exhibit inductances independent of rotor position.
- The complex variable (space phasor) model is credited with a reduction in the number of equations with respect to the d-q model but it operates with complex variables.
- When solving the state space equations, only the d-q model, with real variables, benefits the existing commercial software (Mathematica, Matlab–Simulink, Spice, etc.).
- Both models are very practical in treating the transients and control of symmetrical IMs fed from symmetrical voltage power grids or from PWM converters.
- Easy incorporation of magnetic saturation and rotor skin effect are two additional assets of complex variable and d-q models. The airgap flux density retains a sinusoidal distribution along the circumferential direction.
- Besides the complex variable which enjoys widespread usage, other models (variable transformations), that deal especially with asymmetric supply or asymmetric machine cases have been introduced (for a summary see Reference. [3, 4])

13.4. STEADY-STATE BY THE COMPLEX VARIABLE MODEL

By IM steady-state we mean constant speed and load. For a machine fed from a sinusoidal voltage symmetrical power grid, the phase voltages at IM terminals are

$$V_{a,b,c} = V\sqrt{2} \cdot \cos\left(\omega_1 t - (i-1) \cdot \frac{2\pi}{3}\right); \quad i = 1, 2, 3 \quad (13.39)$$

The voltage space phasor $\overline{V}_s^b$ in random coordinates (from (13.27)) is

$$\overline{V}_s^b = \frac{2}{3} \left(V_a(t) + a V_b(t) + a^2 V_c(t) \right) e^{-j\theta_{cr}} \quad (13.40)$$

From (13.39) and (13.40),

$$\overline{V}_s^b = V\sqrt{2} [\cos(\omega_1 t - \theta_b) + j \sin(\omega_1 t - \theta_b)] \quad (13.41)$$

Only for steady-state,

$$\theta_b = \omega_b t + \theta_0 \quad (13.42)$$

Consequently,

$$\overline{V}_s^b = V\sqrt{2} e^{j[(\omega_1 - \omega_b)t + \theta_0]} \quad (13.43)$$

For steady-state, the current in the space phasor model follows the voltage frequency: $(\omega_1 - \omega_b)$. Steady-state in the state space equations means replacing d/dt with $j(\omega_1 - \omega_b)$.

Using this observation makes Equations (13.30) become

$$\begin{aligned}\bar{V}_{s0} &= R_s \bar{i}_{s0} + j\omega_1 \bar{\Psi}_{s0}; \quad \bar{\Psi}_{s0} = L_{sl} \bar{i}_{s0} + \bar{\Psi}_{m0}; \\ \bar{\Psi}_{r0} &= L_{rl} \bar{i}_{r0} + \bar{\Psi}_{m0}; \quad \bar{i}_{m0} = \bar{i}_{s0} + \bar{i}_{r0} \\ \bar{V}_{r0} &= R_r \bar{i}_{r0} + jS\omega_1 \bar{\Psi}_{r0}; \quad S = \frac{(\omega_1 - \omega_r)}{\omega_r}; \quad \bar{\Psi}_{m0} = L_m \bar{i}_{m0}\end{aligned}\quad (13.44)$$

So the form of space phasor model voltage equations under the steady-state is the same irrespective of the speed of the reference system ω_b .

When ω_b , only the frequency changes of voltages, currents, flux linkages in the space phasor model varies as it is $\omega_1 - \omega_b$.

No wonder this is so, as only Equations (13.44) exhibit the total emf, which should be independent of reference system speed ω_b . S is the slip, a variable well known by now.

Notice that for $\omega_b = \omega_1$ (synchronous coordinates), $d/dt = (\omega_1 - \omega_b) = 0$. Consequently, for synchronous coordinates the steady-state means d.c. variables.

The space phasor diagram of (13.44) is shown in Figure 13.2 for a cage rotor IM.

cage rotor

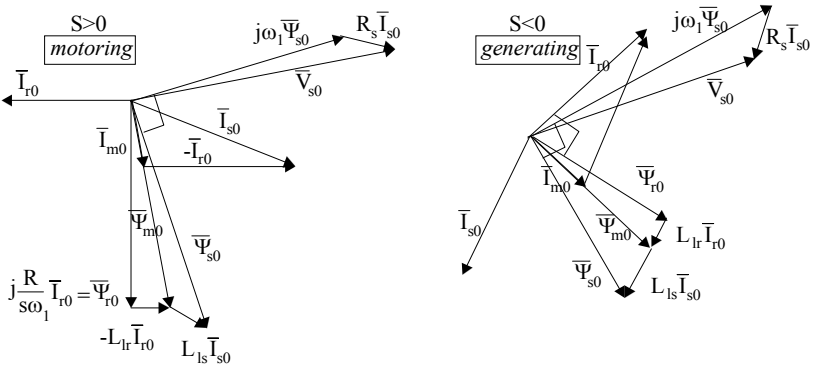


Figure 13.2 Space phasor diagram for steady-state

From the stator space Equations (13.44) the torque (13.31) becomes

$$T_e = \frac{3}{2} p_l \left(j \bar{\Psi}_{r0} \bar{i}_{r0}^* \right) = \frac{3}{2} P_l \Psi_{r0} i_{r0} \quad (13.45)$$

Also, from (13.44),

$$\bar{i}_{r0} = -jS\omega_1 \frac{\bar{\Psi}_{r0}}{R_r} \quad (13.46)$$

With (13.46), alternatively, the torque is

$$T_e = \frac{3}{2} p_1 \frac{\Psi_{r0}^2}{R_r} S \omega_1 \quad (13.47)$$

Solving for Ψ_{r0} in Equations (13.44) lead to the standard torque formula.

$$T_e \approx \frac{3p_1}{\omega_1} \frac{V_s^2 \frac{R_r}{S}}{\left(R_s + C_1 \frac{R_r}{S}\right)^2 + \omega_1^2 (L_{ls} + C_1 L_{lr})^2}; \quad C_1 = 1 + \frac{L_{ls}}{L_m} \quad (13.48)$$

Expression (13.47) shows that, for constant rotor flux space-phasor amplitude, the torque varies linearly with speed as it does in a separately excited d.c. motor. So all steady-state performance may be calculated using the space-phasor model as well.

13.5. EQUIVALENT CIRCUITS FOR DRIVES

Equations (13.30) lead to a general equivalent circuit good for transients, especially in variable speed drives (Figure 13.3).

$$\begin{aligned} \bar{V}_s &= R_s \bar{i}_s + (p + j\omega_b) L_{sl} \bar{i}_s + (p + j\omega_b) \bar{\Psi}_m \\ \bar{V}_r &= R_r \bar{i}_r + (p + j(\omega_b - \omega_r)) L_{rl} \bar{i}_r + (p + j(\omega_b - \omega_r)) \bar{\Psi}_m \end{aligned} \quad (13.49)$$

The reference system speed ω_b may be random, but three particular values have met with rather wide acceptance.

- Stator coordinates: $\omega_b = 0$; for steady-state: $p \rightarrow j\omega_1$
- Rotor coordinates: $\omega_b = \omega_r$; for steady-state: $p \rightarrow jS\omega_1$
- Synchronous coordinates: $\omega_b = \omega_1$; for steady-state: $p \rightarrow 0$

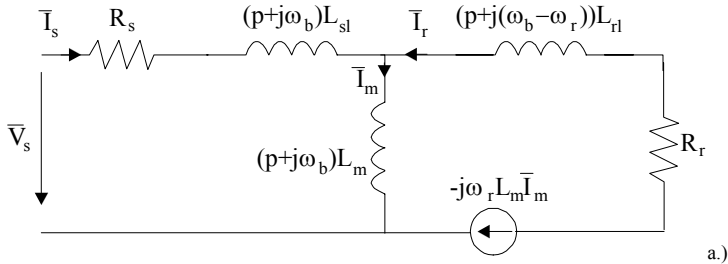


Figure 13.3 The general equivalent circuit a.) and for steady-state b.) (continued)

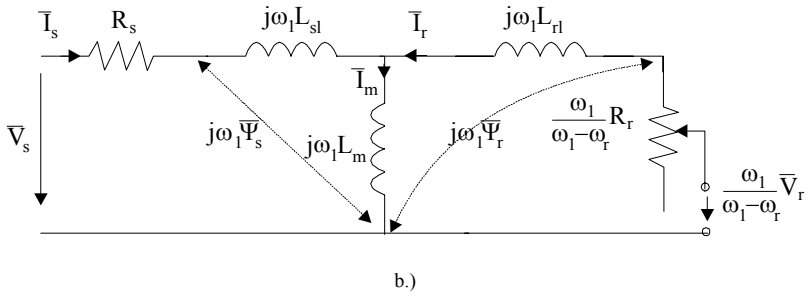


Figure 13.3 (continued)

Also, for steady-state in variable speed drives, the steady-state circuit, the same for all values of ω_b , comes from Figure 13.3a with $p \rightarrow j(\omega_l - \omega_b)$ and Figure 13.3b.

Figure 13.3b shows, in fact, the standard T equivalent circuit of IM for steady-state, but in space phasors and not in phase phasors.

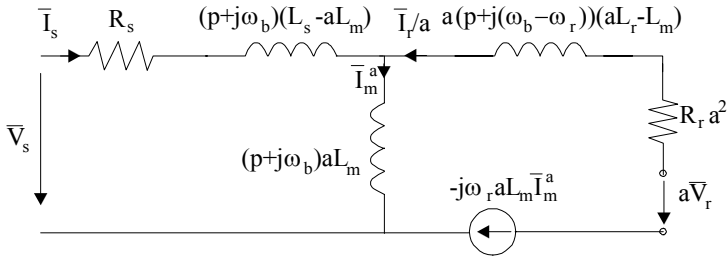


Figure 13.4 Generalized equivalent circuit

A general method to “arrange” the leakage inductances L_{sl} and L_{rl} in various positions in the equivalent circuit consists of a change of variables.

$$\tilde{i}_r^a = \tilde{i}_r / a; \quad \bar{\Psi}_{ma} = aL_m \left(\tilde{i}_s + \tilde{i}_r^a \right) \quad (13.50)$$

Making use of this change of variables in (13.49) yields

$$\begin{aligned} \bar{V}_s &= R_s \tilde{i}_s + (p + j\omega_b)(L_s - aL_m) \tilde{i}_s + (p + j\omega_b) \bar{\Psi}_{ma}^a \\ a\bar{V}_r &= a^2 R_r \tilde{i}_r^a + (p + j(\omega_b - \omega_r))a(aL_r - L_m) \tilde{i}_r^a + (p + j(\omega_b - \omega_r)) \bar{\Psi}_{ma}^a \end{aligned} \quad (13.51)$$

An equivalent circuit may be developed based on (13.51), Figure 13.4.

The generalized equivalent circuit in Figure 13.4 warrants the following comments:

- For $a = 1$, the general equivalent circuit of Figure 13.4 is reobtained and $a\bar{\Psi}_{ma}^a = \bar{\Psi}_m$: the main flux

- For $a = L_m/L_r$ the inductance term in the “rotor section” “disappears”, being moved to the primary section and

$$a \bar{\Psi}_m^a = \frac{L_m}{L_r} L_m \left(\dot{\bar{i}}_s + \dot{\bar{i}}_r \frac{L_r}{L_m} \right) = \frac{L_m}{L_r} \bar{\Psi}_r \quad (13.52)$$

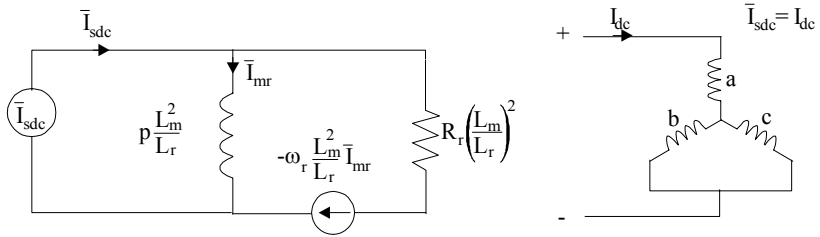
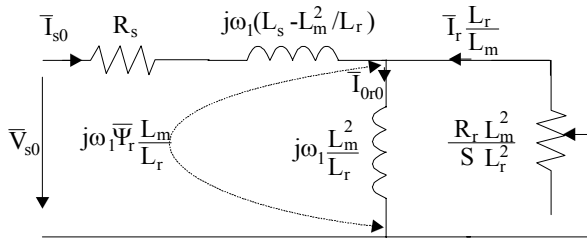
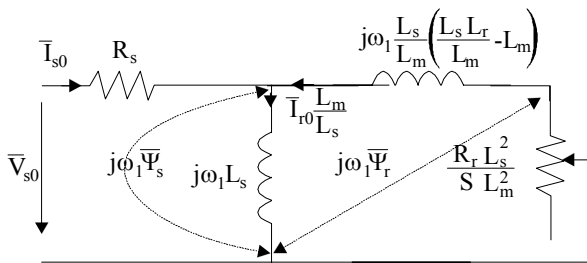


Figure 13.5 Equivalent circuit for dc braking



a.)



b.)

Figure 13.6 Steady-state equivalent circuits

a.) rotor flux oriented b.) stator flux oriented

- For $a = L_s/L_m$, the leakage inductance term is lumped into the “rotor section” and

$$a\bar{\Psi}_m^a = \frac{L_s}{L_m} L_m \left(\bar{i}_s + \bar{i}_r \frac{L_m}{L_s} \right) = \bar{\Psi}_s \quad (13.53)$$

- This type of equivalent circuit is adequate for stator flux orientation control
- For d.c. braking, the stator is fed with d.c. The method is used for variable speed drives. The model for this regime is obtained from Figure 13.4 by using a d.c. current source, $\omega_b = 0$ (stator coordinates, $V_r = 0$, $a = L_m/L_r$)
The result is shown in Figure 13.5.

For steady-state, the equivalent circuits for $a_r = L_m/L_r$ and $a_r = L_s/L_m$ and $V_r=0$ are shown in Figure 13.6.

Example 13.1. The constant rotor flux torque/speed curve.

Let us consider an induction motor with a single rotor cage and constant parameters: $R_s = R_r = 1 \Omega$, $L_{sl} = L_{rl} = 5 \text{ mH}$, $L_m = 200 \text{ mH}$, $\Psi_{r0}' = 1 \text{ Wb}$, $S = 0.2$, $\omega_1 = 2\pi 6 \text{ rad/s}$, $p_1 = 2$, $V_r = 0$. Find the torque, rotor current, stator current, stator and main flux and voltage for this situation. Draw the corresponding space phasor diagram.

Solution

We are going to use the equivalent circuit in Figure 13.6a and the rotor current and torque expressions (13.46 and 13.47):

$$T_e = \frac{3}{2} p_1 \frac{\Psi_{r0}^2}{R_r} S \omega_1 = \frac{3}{2} 2 \frac{1^2}{1} 0.2 \cdot 2\pi 6 = 22.608 \text{ Nm}$$

$$I_{r0} = -S \omega_1 \frac{\Psi_{r0}}{R_r} = -0.2 \cdot 2\pi 6 \frac{1}{1} = -7.536 \text{ A}$$

The rotor current is placed along real axis in the negative direction. The rotor flux magnetization current $\bar{I}_{r0}$ (Figure 13.6a) is

$$\bar{I}_{0r0} = -j \frac{(-I_{r0}) \frac{R_r}{S} \frac{L_m}{L_r}}{\omega_1 \frac{L_m^2}{L_r}} = -j \frac{(-I_{r0}) R_r}{S \omega_1 L_m} = -j \frac{(+7.536) \cdot 1}{0.2 \cdot 2\pi 6 \cdot 0.2} = -j5 \text{ A}$$

The stator current

$$\bar{I}_{s0} = -\bar{I}_{r0} \frac{L_r}{L_m} + \bar{I}_{0r0} = 7.536 \frac{0.205}{0.2} - j5 = 7.7244 - j5.0 \text{ A}$$

The stator flux $\bar{\Psi}_s$ is

$$\bar{\Psi}_{s0} = L_s \bar{I}_{s0} + L_m \bar{I}_{r0} = 0.205(7.7244 - j5.0) + 0.2(-7.536) = 0.076 - j1.025$$

The airgap flux $\bar{\Psi}_m$ is

$$\bar{\Psi}_{m0} = L_m (\bar{I}_{s0} + \bar{I}_{r0}) = 0.2(7.7244 - j5.0 - 7.536) = 0.03768 - j1.0$$

The rotor flux is

$$\bar{\Psi}_{r0} = \bar{\Psi}_{m0} + L_{rl} \bar{I}_{r0} = 0.03768 - j1.0 + 0.005(-7.536) = -j1.0 \text{ (as expected)}$$

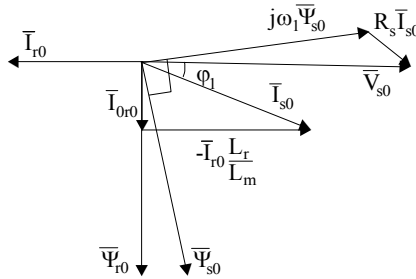


Figure A The space phasor diagram

The voltage V_{s0} is:

$$V_{s0} = j\omega_l \bar{\Psi}_{s0} + R_s \bar{I}_{s0} = j2\pi 6(0.076 - j1.025) + 1(7.7244 - j5.0) = 46.346 - j2.136$$

The corresponding space phasor diagram is shown in [Figure A](#).

13.6. ELECTRICAL TRANSIENTS WITH FLUX LINKAGES AS VARIABLES

Equations (13.30) may be transformed by changing the variables through currents elimination.

$$\begin{aligned} \dot{\bar{i}}_s &= \sigma^{-1} \left[\frac{\bar{\Psi}_s}{L_s} - \bar{\Psi}_r \frac{L_m}{L_s L_r} \right]; \quad \sigma = 1 - \frac{L_m^2}{L_s L_r} \\ \dot{\bar{i}}_r &= \sigma^{-1} \left[\frac{\bar{\Psi}_r}{L_r} - \bar{\Psi}_s \frac{L_m}{L_s L_r} \right] \end{aligned} \quad (13.54)$$

$$\begin{aligned} \tau_s' \frac{d\bar{\Psi}_s}{dt} + (1 + j \cdot \omega_b \cdot \tau_s') \cdot \bar{\Psi}_s &= \tau_s' \bar{V}_s + K_r \bar{\Psi}_r \\ \tau_r' \frac{d\bar{\Psi}_r}{dt} + (1 + j \cdot (\omega_b - \omega_r) \cdot \tau_r') \cdot \bar{\Psi}_r &= \tau_r' \bar{V}_r + K_s \bar{\Psi}_s \end{aligned} \quad (13.55)$$

with

$$\begin{aligned}
K_s &= \frac{L_m}{L_s}; & K_r &= \frac{L_m}{L_r} \\
\tau_s' &= \tau_s \cdot \sigma; & \tau_r' &= \tau_r \cdot \sigma \\
\tau_s &= \frac{L_s}{R_s}; & \tau_r &= \frac{L_r}{R_r}
\end{aligned} \tag{13.56}$$

By electrical transients we mean constant speed transients. So both ω_b and ω_r are considered known. The inputs are the two voltage space phasors $\bar{V}_s$ and $\bar{V}_r$ and the outputs are the two flux linkage space phasors, $\bar{\Psi}_s$ and $\bar{\Psi}_r$.

The structural diagram of Equations (13.55) is shown in Figure 13.7.

The transient behavior of stator and rotor flux linkages as complex variables, at constant speed ω_b and ω_r , for standard step or sinusoidal voltages $\bar{V}_s$, $\bar{V}_r$ signals has analytical solutions. Finally, the torque transients also have analytical solutions as

$$T_e = \frac{3}{2} p_l \operatorname{Re}(j \Psi_s i_s^*) \tag{13.57}$$

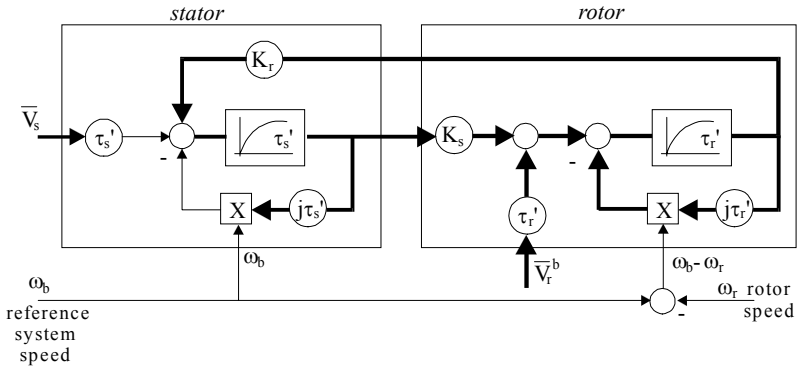


Figure 13.7 IM space-phasor diagram for constant speed

The two complex eigenvalues of (13.55) are obtained from

$$\begin{vmatrix} \tau_s' p + 1 + j \omega_b \tau_s' & -K_r \\ -K_s & \tau_r' p + 1 + j(\omega_b - \omega_r) \tau_r' \end{vmatrix} = 0 \tag{13.58}$$

As expected, the eigenvalues $p_{1,2}$ depend on the speed of the motor and on the speed of the reference system ω_b .

Equation (13.58) may be put in the form

$$\begin{aligned}
& p^2 \tau_s' \tau_r' + p[(\tau_s' + \tau_r') + j \tau_s' \tau_r' (2\omega_b - \omega_r)] - K_s K_r + \\
& + (1 + j \cdot \omega_b \cdot \tau_s')(1 + j \cdot (\omega_b - \omega_r) \cdot \tau_r') = 0
\end{aligned} \tag{13.58'}$$

In essence, in-rush current and torque (at zero speed), for example, has a rather straightforward solution through the knowledge of eigenvalues, with $\omega_r = 0 = \omega_b$.

$$p^2 \tau_s' \tau_r' + p(\tau_s' + \tau_r') - K_s K_r + 1 = 0$$

$$(p_{1,2})_{\omega_r = \omega_b = 0} = \frac{-(\tau_s' + \tau_r') \pm \sqrt{(\tau_s' + \tau_r')^2 - 4\tau_s' \tau_r' (-K_s K_r + 1)}}{2\tau_s' \tau_r'} \quad (13.59)$$

The same capability of yielding analytical solutions for transients is claimed by the spiral vector theory. [5]

For constant amplitude stator or rotor flux conditions

$$\left(\frac{d\bar{\Psi}_s}{dt} = j(\omega_1 - \omega_b)\bar{\Psi}_s \text{ or } \frac{d\bar{\Psi}_r}{dt} = j(\omega_1 - \omega_b)\bar{\Psi}_r ; \omega_1 - \text{stator frequency} \right)$$

Equations (13.55) are left with only one complex eigenvalue. Even a simpler analytical solution for electrical transients is feasible for constant stator and rotor flux conditions, so typical in fast response modern vector control drives.

Also, at least at zero speed, the eigenvalue with voltage supply is about the same for stator or for rotor supply.

The same equations may be expressed with $\bar{i}_s$ and $\bar{\Psi}_r$ as variables, by simply putting $a = L_m/L_r$ and by eliminating $\bar{i}_r$ from (13.51) with (13.50).

13.7. INCLUDING MAGNETIC SATURATION IN THE SPACE PHASOR MODEL

To incorporate magnetic saturation easily into the space phasor model (13.49), we separate the leakage saturation from main flux path saturation with pertinent functions obtained *a priori* from tests or from field solutions (FEM).

$$L_{sl} = L_{sl}(\bar{i}_s), \quad L_{lr} = L_{lr}(\bar{i}_r), \quad \bar{\Psi}_m = L_m(\bar{i}_m)\bar{i}_m \quad (13.60)$$

$$|\bar{i}_m| = |\bar{i}_s + \bar{i}_r| \quad (13.61)$$

Let us consider the reference system oriented along the main flux $\bar{\Psi}_m$, that is, $\bar{\Psi}_m = \Psi_m$, and eliminate the rotor current, maintaining Ψ_m and $\bar{i}_s$ as variables.

$$\begin{aligned} \bar{V}_s &= [R_s + (p + j\omega_b)L_{sl}(\bar{i}_s)] \cdot \bar{i}_s + (p + j\omega_b) \cdot \bar{\Psi}_m \\ \bar{V}_r &= [R_r + (p + j(\omega_b - \omega_r))L_{lr}(\bar{i}_m - \bar{i}_s)] \cdot (\bar{i}_m - \bar{i}_s) + (p + j(\omega_b - \omega_r)) \cdot \bar{\Psi}_m \end{aligned} \quad (13.62)$$

$$i_m = \frac{\Psi_m}{L_m(i_m)} \quad (13.63)$$

We may add the equation of motion,

$$\frac{Jp\omega_r}{P_1} = \frac{3}{2}p_1 \text{Re}(\dot{\Psi}_m \dot{i}_s^*) - T_{\text{load}} \quad (13.64)$$

Provided the magnetization curves $L_{sl}(i_s)$, $L_{lr}(i_r)$ are known, Equations (13.62)–(13.64) may be solved only by numerical methods after splitting Equations (13.62) along d and q axis.

For steady-state, however, it suffices that in the equivalent circuits, L_m is made a function of i_m , L_{sl} of i_{s0} and $L_{lr}(i_{r0})$ (Figure 13.8). This is only in synchronous coordinates where steady-state means dc variables.

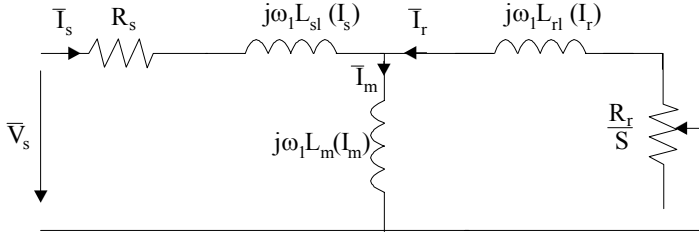


Figure 13.8 Standard equivalent circuit for steady-state leakage and main flux path saturation

In reality, both in the stator and rotor, the magnetic fields are a.c. at frequency ω_1 and $S\omega_1$. So, in fact, in Figure 13.8, the transient (a.c.), inductances should be used.

$$\begin{aligned} L_{ls}^t(i_s) &= L_{ls}(i_s) + \frac{\partial L_{ls}}{\partial i_s} i_s < L_{ls}(i_s) \\ L_{lr}^t(i_r) &= L_{lr}(i_r) + \frac{\partial L_{lr}}{\partial i_r} i_r < L_{lr}(i_r) \\ L_m^t(i_m) &= L_m(i_m) + \frac{\partial L_m}{\partial i_m} i_m < L_m(i_m) \end{aligned} \quad (13.65)$$

Typical curves are shown in Figure 13.9.

As the transient (a.c.) inductances are even smaller than the normal (d.c.) inductances, the machine behavior at high currents is expected to show further increased currents.

Furthermore, as shown in Chapter 6, the leakage flux circumferential flux lines at high currents influence the main (radial) flux and contribute to the resultant flux in the machine core. The saturation in the stator is given by the stator flux Ψ_s and in the rotor by the rotor flux for high levels of currents.

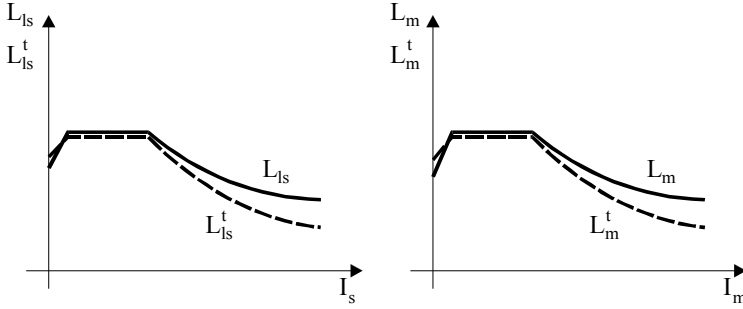


Figure 13.9 Leakage and main flux inductances versus current

So, for large currents, it seems more appropriate to use the equivalent circuit with stator and rotor flux shown (Figure 13.10). However, two new variable inductances, L_{si} and L_{ri} , are added. L_g refers only to the airgap only. Finally the stator and rotor leakage inductances are related only to end connections and slot volume: L_{ls}^e , L_{lr}^e . [6]

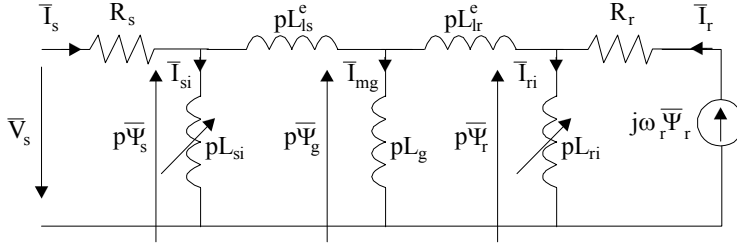


Figure 13.10 Space-phasor equivalent circuit with stator and rotor core saturation included (stator coordinates)

As expected, the functions $\Psi_s(i_{si}) = L_{si}i_{si}$ and $\Psi_r(i_{ri}) = L_{ri}i_{ri}$ have to be known together with L_{ls}^e , L_{lr}^e , R_s , R_r , L_g which are hereby considered constant.

In the presence of skin effect, L_{lr}^e and R_r are functions of slip frequency $\omega_{sr} = \omega_1 - \omega_r$. Furthermore we should notice that it is not easy to measure all parameters in the equivalent circuit on Figure 13.10. It is, however, possible to calculate them either by sophisticated analytical or through FEM models. Depending on the machine design and load, the relative importance of the two variable inductances L_{si} and L_{ri} may be notable or negligible.

Consequently, the equivalent circuit may be simplified. For example, for heavy loads the rotor saturation may be ignored and thus only L_{si} remains. Then L_{si} and L_g in parallel may be lumped into an equivalent variable inductance L_{ms} and L_{ls}^e and L_{lr}^e into the total constant leakage inductance of the machine L_l . Then the equivalent circuit of Figure 13.10 degenerates into the one of Figure 13.11.

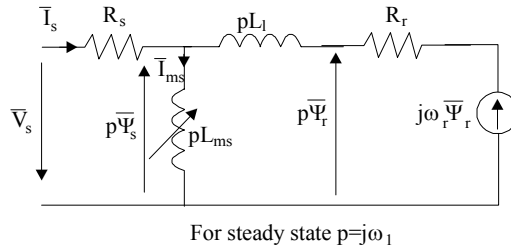


Figure 13.11 Space-phasor equivalent circuit with stator saturation included (stator coordinates)

When high currents occur, during transients in electric drives, the equivalent circuit of Figure 13.11 indicates severe saturation while the conventional circuit (Figure 13.8) indicates moderate saturation of the main path flux and some saturation of the stator leakage path.

So when high current (torque) transients occur, the real machine, due to the L_{ms} reduction, produces torque performance quite different from the predictions by the conventional equivalent circuit (Figure 13.8), up to 2 to 2.5 p.u. current, however, the differences between the two models are negligible.

In IMs designed for extreme saturation conditions (minimum weight), models like that in Figure 13.10 have to be used, unless FEM is applied.

13.8. SATURATION AND CORE LOSS INCLUSION INTO THE STATE-SPACE MODEL

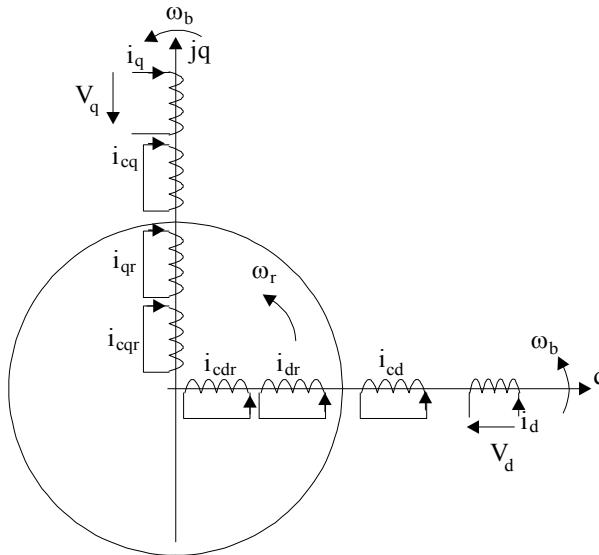


Figure 13.12 d-q model with stator and core loss windings

To include the core loss in the space phasor model of IMs, we assume that the core losses occur, both in the stator and rotor, into equivalent orthogonal windings: $c_d - c_q$, $c_{dr} - c_{qr}$ (Figure 13.12).

Alternatively, when rotor core loss is neglected (cage rotor IMs), the $c_{dr} - c_{qr}$ windings account for the skin effect via the double-cage equivalence principle.

Let us consider also that the core loss windings are coupled to the other windings only by the main flux path.

The space phasor equations are thus straightforward (by addition).

$$\begin{aligned}
 R_s \bar{i}_s - \bar{V}_s &= -\frac{d\bar{\Psi}_s}{dt} - j\omega_b \bar{\Psi}_s; \quad \bar{\Psi}_s = \bar{\Psi}_m + L_{ls} \bar{i}_s \\
 R_{cs} \bar{i}_{cs} &= -\frac{d\bar{\Psi}_{cs}}{dt} - j\omega_b \bar{\Psi}_{cs}; \quad \bar{\Psi}_{cs} = \bar{\Psi}_m + L_{lcs} \bar{i}_{cs} \\
 R_r \bar{i}_r - \bar{V}_r &= -\frac{d\bar{\Psi}_r}{dt} - j(\omega_b - \omega_r) \bar{\Psi}_r; \quad \bar{\Psi}_r = \bar{\Psi}_m + L_{lr} \bar{i}_r \\
 R_{cr} \bar{i}_{cr} &= -\frac{d\bar{\Psi}_{cr}}{dt} - j(\omega_b - \omega_r) \bar{\Psi}_{cr}; \quad \bar{\Psi}_{cr} = \bar{\Psi}_m + L_{lcr} \bar{i}_{cr}
 \end{aligned} \tag{13.66}$$

$$\bar{i}_m = (\bar{i}_s + \bar{i}_{cs} + \bar{i}_r + \bar{i}_{cr}) \quad \bar{\Psi}_m = L_m (\bar{i}_m) \bar{i}_m \tag{13.67}$$

The airgap torque now contains two components, one given by the stator current and the other one (braking) given by the stator core losses.

$$T_e = \frac{3}{2} p_l \operatorname{Re} [j(\bar{\Psi}_s \bar{i}_s^* + \bar{\Psi}_{cs} \bar{i}_{cs}^*)] = \frac{3}{2} p_l \operatorname{Re} [j\bar{\Psi}_m (\bar{i}_s^* + \bar{i}_{cs}^*)] \tag{13.68}$$

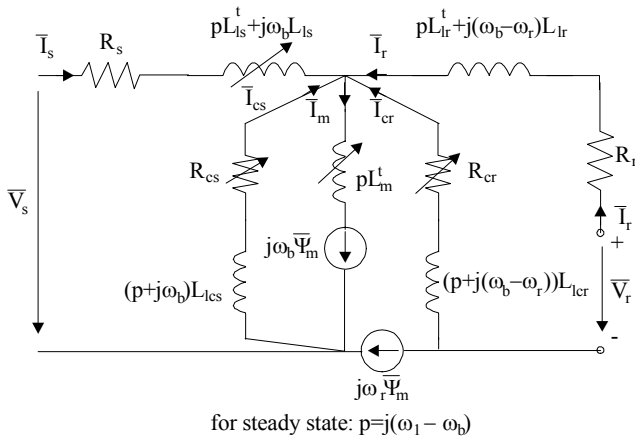


Figure 13.13 Space phasor T equivalent circuit with saturation and rotor core loss (or rotor skin effect)

Equations (13.66) and (13.67) lead to a fairly general equivalent circuit when we introduce the transient magnetization inductance L_{mt} (13.65) and consider separately main flux and leakage paths saturation (Figure 13.13).

The apparently involved equivalent circuit in Figure 13.13 is fairly general and may be applied for many practical cases such as

- The reference system may be attached to the stator, $\omega_b = 0$ (for cage rotor and large transients), to the rotor (for the doubly fed IM), or to stator frequency $\omega_b = \omega_1$ for electric drives transients.
- To simplify the solving of Equations (13.66) and (13.67) the reference system may be attached to the main flux space phasor: $\bar{\Psi}_m = \Psi_m(i_m)$ and eventually using $i_m, i_{cs}, i_{cr}, i_r, \omega_r$ as variables with $\bar{i}_s$ as a dummy variable. [7,8] As expected, d-q decomposition is required.
- For a cage rotor with skin effect (medium and large power motors fed from the power grid directly), we should simply make $V_r = 0$ and consider the rotor core loss winding (with its equations) as the second (fictitious) rotor cage.

Example 13.2. Saturation and core losses

Simulation results are presented in what follows. The motor constant parameters are: $R_s = 3.41\Omega$, $R_r = 1.89\Omega$, $L_{sl} = 1.14 \cdot 10^{-2}H$, $L_{rl} = 0.9076 \cdot 10^{-2}H$, $J = 6.25 \cdot 10^{-3}Kgm^2$. The magnetization curve $\Psi_m(i_m)$ is shown in Figure 13.14 together with core loss in the stator at 50 Hz.

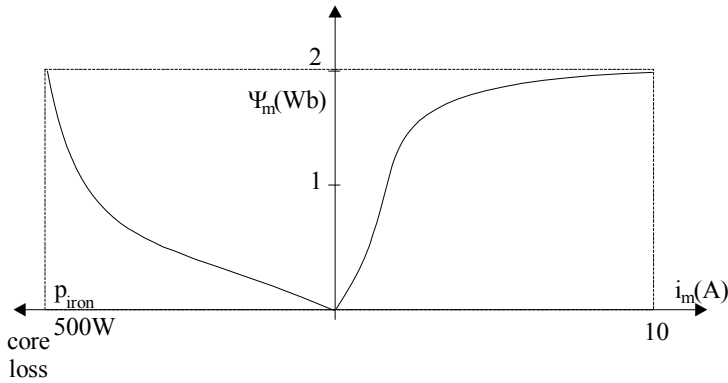


Figure 13.14 Magnetization curve and stator core loss

The stator core loss resistance R_{cs}

$$R_{cs} = \frac{3}{2} \frac{\omega_1^2 \Psi_m^2}{P_{iron}} \approx \frac{3}{2} \frac{(2\pi 50)^2 \cdot 2^2}{500} = 1183.15\Omega$$

Considering that $L_{lcs} \cdot \omega_1 = R_{cs}$; $L_{lcs} = 1183.15 / (2\pi 50) = 3.768 H$.

For steady-state and synchronous coordinates ($d/dt = 0$), Equations (13.66) become

$$\begin{aligned} R_s \vec{i}_{s0} - \vec{V}_{s0} &= -j\omega_l \vec{\Psi}_{s0}; \quad V_{s0} = 380\sqrt{2}e^{j\delta_0} - V \text{ d.c.} \\ R_{cs} \vec{i}_{cs0} &= -j\omega_l \vec{\Psi}_{cs0}; \quad \vec{i}_{m0} = \vec{i}_{s0} + \vec{i}_{cs0} + \vec{i}_{r0} \\ R_r \vec{i}_{r0} &= -jS\omega_l \vec{\Psi}_{r0} \end{aligned} \quad (13.69)$$

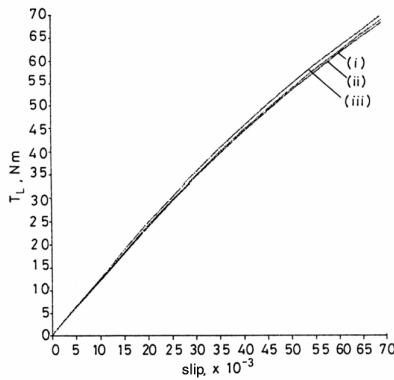
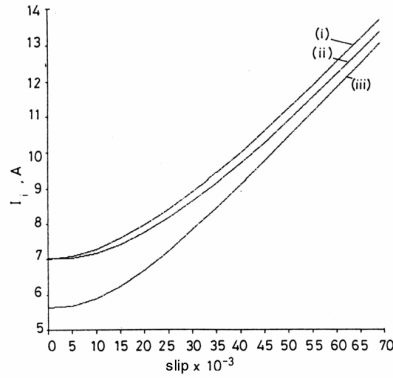


Figure 13.15 Steady-state phase current and torque versus slip
(i) saturation and core loss considered (ii) saturation considered, core loss neglected
(iii) no saturation, no core loss

For given values of slip S , the values of stator current $i_{s0}/\sqrt{2}$ (RMS/phase) and the electromagnetic torque are calculated for steady-state making use of (13.69) and the flux/current relationships (13.66) and (13.67) and [Figure 13.14](#).

The results are given in Figure 13.15a and b. There are notable differences in stator current due to saturation. The differences in torque are rather small. Efficiency–power factor product and the magnetization current versus slip are shown in Figure 13.16a and b.

Again, saturation and core loss play an important role in reducing the efficiency–power factor although core loss itself tends to increase the power factor while it tends to reduce the efficiency.

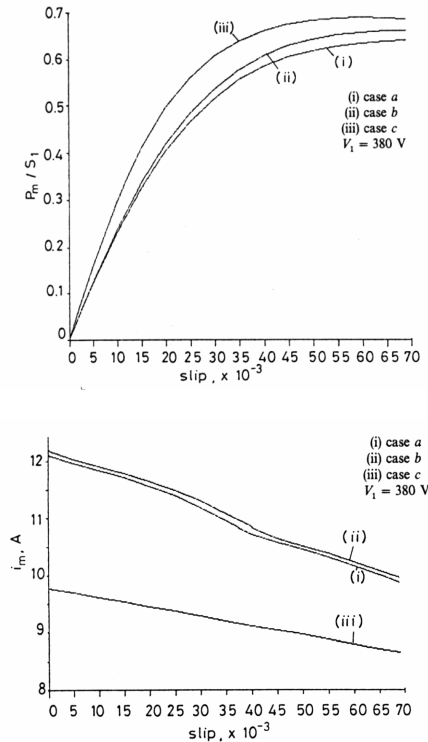


Figure 13.16 Efficiency–power factor product and magnetization current versus slip

The reduction of magnetization current i_m with slip is small but still worth considering.

Two transients have been investigated by solving (13.66) and (13.67) and the motion equations through the Runge–Kutta–Gill method.

1. Sudden 40% reduction of supply voltage from 380 V per phase (RMS), steady-state constant load corresponding to $S = 0.03$.
2. Disconnection of the loaded motor at $S = 0.03$ for 10 ms and reconnection (at $\delta_0 = 0$, though any phasing could be used). The load is constant again.

The transients for transient 1 are shown in Figure 13.17a, b, c.

Some influence of saturation and core loss occurs in the early stages of the transients, but in general the influence of them is moderate because at reduced voltage, saturation influence diminishes considerably.

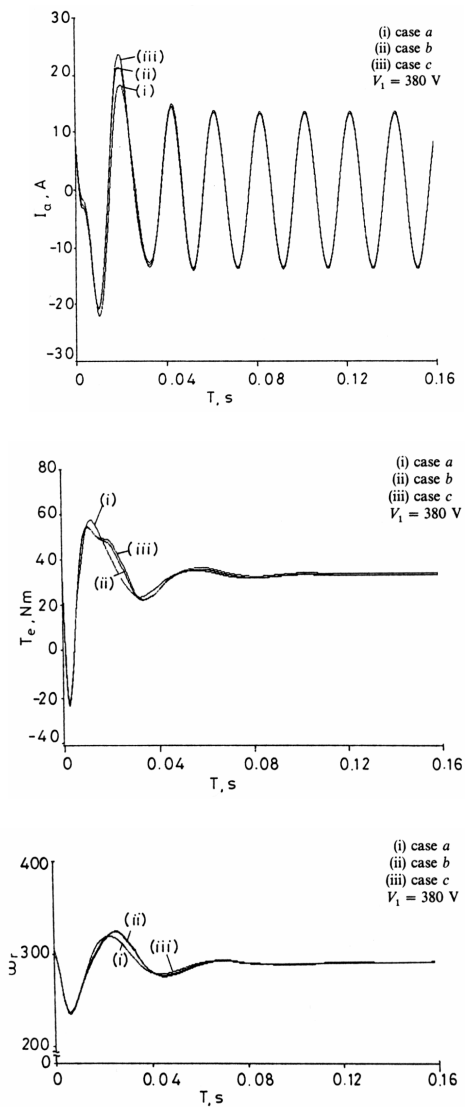


Figure 13.17 Sudden 40% voltage reduction at $S = 0.03$ constant load
a.) stator phase current, b.) torque, c.) speed

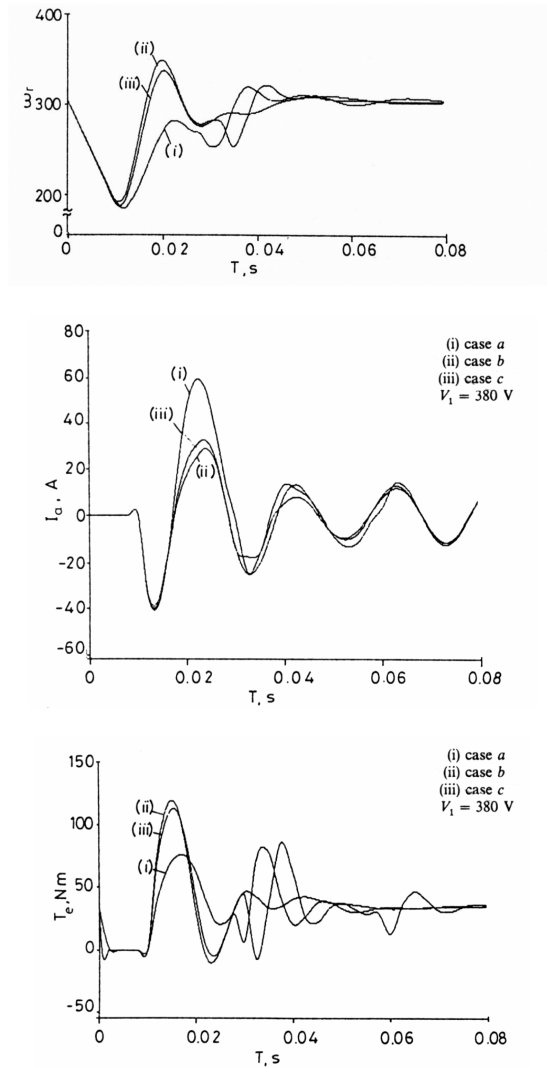


Figure 13.18 Disconnection–reconnection transients at high voltage: $V_{\text{phase}} = 380\text{V (RMS)}$

The second transient, occurring at high voltage (and saturation level), causes more important influences of saturation (and core loss) as shown in Figure 13.18a, b, c.

The saturation and core loss lead to higher current peaks, but apparently for low torque and speed transients.

In high-performance variable-speed drives, high levels of saturation may be inflicted to increase the torque transient capabilities of IM. Vector control

detuning occurs and it has to be corrected. Also, in very precise torque control drives, such second order phenomena are to be considered. Skin effect also influences the transients of IMs and the model on [Figure 13.13](#) can handle it for cage rotor IMs directly as shown earlier in this paragraph.

13.9. REDUCED ORDER MODELS

The rather involved d-q (space phasor) model with skin effect and saturation may be used directly to investigate the induction machine transients. More practical (simpler) solutions have also been proposed. They are called reduced order models. [9,10,11]

The complete model has, for a single cage, a fifth order in d-q writing and a third order in complex variable writing ($\bar{\Psi}_s$, $\bar{\Psi}_r$, ω_r).

In general, the speed transients are considered slow, especially with inertia or high loads, while stator flux $\bar{\Psi}_s$ and rotor flux $\bar{\Psi}_r$ transients are much faster for voltage-source supply.

The intuitive way to obtain reduced models is to ignore fast transients, of the stator, $\frac{d\bar{\Psi}_s}{dt} = 0$ (in synchronous coordinates) or stator and rotor transients

$\left(\frac{d\bar{\Psi}_s}{dt} = \frac{d\bar{\Psi}_r}{dt} = 0 \right)$ (in synchronous coordinates) and up date the speed by solving the motion equation by numerical methods.

This way, third order $\left(\frac{d\bar{\Psi}_s}{dt} = 0 \right)$ or first order $\left(\frac{d\bar{\Psi}_s}{dt} = \frac{d\bar{\Psi}_r}{dt} = 0 \right)$ models

are obtained. The problem is that the results of such order obscure reductions inevitably fast transients. Electric (supply frequency) torque transients (due to rotor flux transients) during starting, may still be visible in such models, but for too long a time interval during starting in comparison with the reality.

So there are two questions about model order reduction.

- What kind of transients are to be used
- What torque transients have to be preserved

In addition to this, small and large power IMs seem to require different reduced orders to produce practical results in simulating a group of IMs when the computation time is critical.

13.9.1. Neglecting stator transients

In this case, in synchronous coordinates, the stator flux derivative is considered zero (Equations (13.55)).

$$\begin{aligned}
0 + (1 + j\omega_1\tau_s')\bar{\Psi}_s &= \tau_s'\bar{V}_s + K_r\bar{\Psi}_r \\
\tau_r'\frac{d\bar{\Psi}_r}{dt} + (1 + j(\omega_1 - \omega_r))\bar{\Psi}_r &= \tau_r'\bar{V}_r + K_s\bar{\Psi}_s \\
T_e = -\frac{3}{2}p_1 \operatorname{Re}(\bar{j}\bar{\Psi}_r\bar{i}_r^*); \bar{i}_r^* &= \sigma^{-1}\left(\frac{\bar{\Psi}_r}{L_r} - \bar{\Psi}_s\frac{L_m}{L_sL_r}\right) \\
\frac{J}{p_1}\frac{d\omega_r}{dt} &= T_e - T_{\text{load}}
\end{aligned} \tag{13.70}$$

With two algebraic equations, there are three differential ones (in real d-q variables).

For cage rotor IMs ,

$$\bar{\Psi}_s = \frac{\tau_s'\bar{V}_s + K_r\bar{\Psi}_r}{1 + j\omega_1\tau_s'} \tag{13.71}$$

$$\frac{d\bar{\Psi}_r}{dt} = \left[-(1 + j(\omega_1 - \omega_r)) + \frac{K_sK_r}{1 + j\omega_1\tau_s'} \right] \frac{\bar{\Psi}_r}{\tau_r'} + \frac{\tau_s'\bar{V}_s \cdot K_s}{(1 + j\omega_1\tau_s')\tau_r'} \tag{13.72}$$

$$\begin{aligned}
T_e = \frac{3}{2}p_1 \operatorname{Im}\left[\sigma^{-1}\bar{\Psi}_r\bar{\Psi}_s^* \frac{L_m}{L_sL_r}\right] &= \frac{3}{2}p_1 \frac{\sigma^{-1}L_m}{L_sL_r} \operatorname{Im}\left[\bar{\Psi}_r \frac{\tau_s'\bar{V}_s^* + K_r\bar{\Psi}_r^*}{1 - j\omega_1\tau_s'}\right] \\
\frac{d\omega_r}{dt} &= \frac{p_1}{J}(T_e - T_{\text{load}})
\end{aligned} \tag{13.73}$$

Fast (at stator frequency) transient torque pulsations are absent in this third-order model (Figure 13.8b).

The complete model and third-order model for a motor with $L_s = 0.05H$, $L_r = 0.05H$, $L_m = 0.0474H$, $R_s = 0.29\Omega$, $R_r = 0.38\Omega$, $\omega_1 = 100\pi\text{rad/s}$, $V_s = 220\sqrt{2}\text{ V}$, $p_1 = 2$, $J = 0.5\text{ Kg m}^2$ yields, for no-load, starting transient results as shown in Figure 13.19. [11]

It is to be noted that steady-state torque at high slips (Figure 13.19a) falls into the middle of torque pulsations, which explains why calculating no-load starting time with steady-state torque produces good results.

A second-order system may be obtained by considering only the amplitude transients of $\bar{\Psi}_r$ in (13.73) [11], but the results are not good enough in the sense that the torque fast (grid frequency) transients are present during start up at higher speed than for the full model.

Modified second-order models have been proposed to improve the precision of torque results [11] (Figure 13.19c), but the results are highly dependent on motor parameters and load during starting. So, in fact, for starting transients when the torque pulsations (peaks) are required with precision, model reduction may be exercised with extreme care.

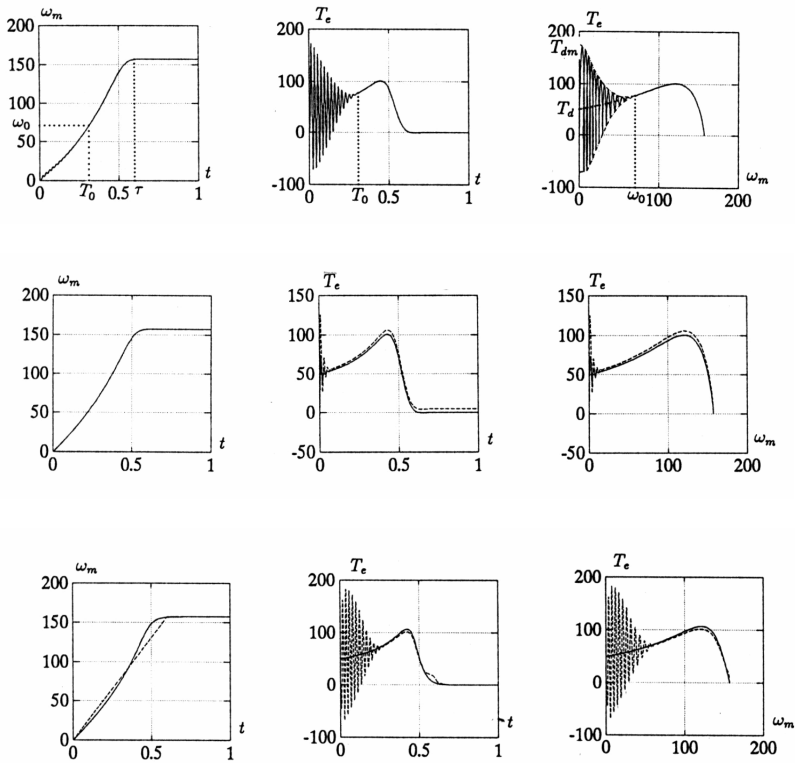


Figure 13.19 Starting transients [11]

- a.) complete model (5th order),
b.) third model (stator transients neglected), c.) modified second order

The presence of leakage saturation makes the above results more of academic interest.

13.9.2. Considering leakage saturation

As shown in Chapter 9, the leakage inductance ($L_{sc} = L_{sl} + L_{rl} = L_l$) decreases with current. This reduction is accentuated when the rotor has closed slots and more moderate when both, stator and rotor, have semiclosed slots (Figure 13.20).

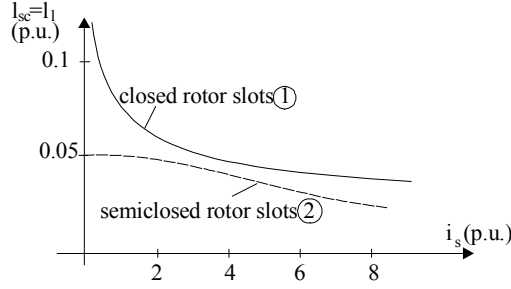


Figure 13.20 Typical leakage inductance versus stator current for semiclosed stator slots

The calculated (or measured) curves of Figure 13.20 may be fitted with analytical expressions of RMS rotor current such as [12]

$$X_1 = \omega_1 L_1 = K_1 + K_2 \left(\frac{I_r}{\sqrt{2}} \right)^{-K} \quad \text{for curve 1} \quad (13.74)$$

$$X_1 = \omega_1 L_1 = K_3 - K_4 \tanh(K_5 I_r^3 - K_6) + K_7 \cos(K_8 I_r) \quad \text{for curve 2} \quad (13.75)$$

During starting on no-load the stator current envelope $I_{sp}(t)$ varies approximately as [12]

$$I_{sp} \approx \frac{Q_1 I_{rk}}{\sqrt{3 \left(1 + \sqrt{\frac{t}{t_{k0}}} + \left(\frac{t}{t_{k0}} \right)^2 \right)}} \quad (13.76)$$

Q_1 reflects the effect of point on wave connection when nonsymmetrical switching of supply is considered to reduce torque transients (peaks) [13]

$$Q_1 \approx \sqrt{2 + \left(\frac{X_1(S_k)}{R_s + R_r} \right) \cdot (\cos^2 \alpha + \sin^2(\alpha + \beta))} \quad (13.77)$$

α —point on wave connection of first phase to phase (b-c) voltage.

β —delay angle in connecting the third phase.

(It has been shown that minimum current (and torque) transients are obtained for $\alpha = \pi/2$ and $\beta = \pi/2$. [14])

As the steady-state torque gives about the same starting time as the transient one, we may use it in the motion equation (for no-load).

$$\frac{J}{p_1} \frac{d\omega_r}{dt} = \frac{3V_{ph}^2 R_r}{\omega_1 p_1} \cdot \frac{1}{\left(R_s + \frac{R_r}{S} \right)^2 S + \omega_1 L_1^2 S} = T_e \quad (13.78)$$

We may analytically integrate this equation with respect to the time up to the breakdown point: S_K slip, noting that $\frac{d\omega_r}{dt} = -\omega_l \frac{dS}{dt}$.

$$t_{K0} = \frac{J\omega_l^2}{3V_{ph}^2 R_r^2} \cdot \left[\frac{1}{2} (R_s^2 + (\omega_l L_l (S_{K0}))^2) (1 - S_{K0}^2) + 2R_s R_r (1 - S_{K0}) + R_r^2 \ln \left(\frac{1}{S_{K0}} \right) \right] \quad (13.79)$$

with
$$S_{K0} = \frac{R_r}{\sqrt{R_s^2 + (\omega_l L_l (S_{K0}))^2}} \quad (13.80)$$

On the other hand, the base current I_{rK} is calculated at standstill.

$$I_{rK} = \frac{V_{ph}}{\sqrt{(R_s + R_r)^2 + (\omega_l L_l (1))^2}} \quad (13.81)$$

Introducing L_l as function of I_r from (13.74) or (13.75) into (13.81) we may solve it numerically to find I_{rK} .

Now we may calculate the time variation of I_{sp} (the current envelope) versus time during no-load starting. The current envelope compares favourably with complete model results and with test results. [12]

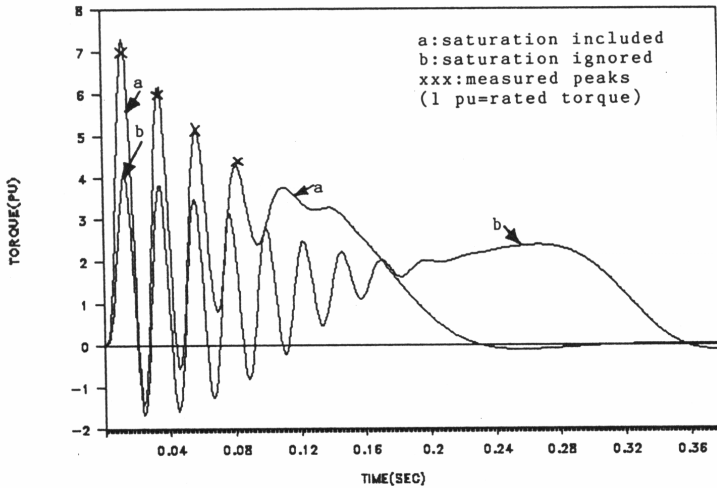


Figure 13.21 Torque transients during no-load starting

As the current envelope varies with time, so does the leakage inductance $L_l(I_{sp})$ and, finally, the torque value in (13.78) is calculated as a function of time through its peak values, since from Equation (13.78) we may calculate slip (S) versus time.

Sample results in Figure 13.21 [12] show that the approximation above is really practical and that neglecting the leakage saturation leads to an underestimation of torque peaks by more than 40%!

13.9.3. Large machines: torsional torque

Starting of large induction machines causes not only heavy starting currents but also high torsional torque due to the large masses of the motor and load which may exhibit a natural torsional frequency in the range of electric torque oscillatory components.

Sudden voltage switching (through a transformer or star/delta), low leakage time constants τ_s' and τ_r' , and a long starting interval cause further problems in starting large inertia loads in large IMs. The layout of such a system is shown in Figure 13.22.

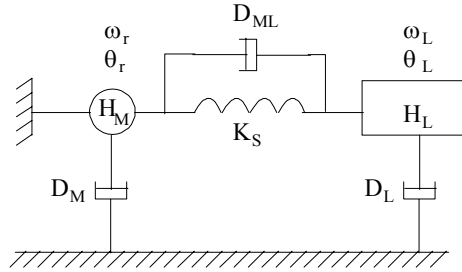


Figure 13.22 Large IM with inertia load

The mechanical equations of the rotating masses in Figure 13.22, with elastic couplings, is straightforward. [15]

$$\begin{bmatrix} \dot{\omega}_m \\ \dot{\theta}_m \\ \dot{\omega}_L \\ \dot{\theta}_L \end{bmatrix} = \begin{bmatrix} -\frac{(D_M + D_{ML})}{2H_M} & -\frac{K_S \omega_0}{2H_M} & \frac{D_{ML}}{2H_M} & \frac{K_S \omega_0}{2H_M} \\ 1 & 0 & 0 & 0 \\ -\frac{D_{ML}}{2H_M} & \frac{K_S \omega_0}{2H_L} & -\frac{(D_L + D_{HL})}{2H_L} & -\frac{K_S \omega_0}{2H_L} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \omega_m \\ \theta_m \\ \omega_L \\ \theta_L \end{bmatrix} \quad (13.82)$$

ω_0 – base speed, H_M , H_L – inertia in seconds, D_{HL} , D_{ML} , D_M in p.u./rad/s, K_s in p.u./rad.

The rotor d-q model Equations (13.34), with single rotor cage, in p.u. (relative units) is, after a few manipulations,

$$\begin{vmatrix} x_s & 0 & x_m & 0 \\ 0 & x_s & 0 & x_m \\ x_m & 0 & x_r & 0 \\ 0 & x_m & 0 & x_r \end{vmatrix} \cdot \frac{1}{\omega_0} \frac{d}{dt} \begin{vmatrix} i_q \\ i_d \\ i_{qr} \\ i_{dr} \end{vmatrix} = \begin{vmatrix} -r_s & -\frac{\omega_m x_s}{\omega_0} & 0 & -\frac{\omega_m x_m}{\omega_0} \\ \frac{\omega_m x_s}{\omega_0} & -r_s & \frac{\omega_m x_m}{\omega_0} & 0 \\ 0 & -Sx_m & -r_r & -Sx_r \\ 0 & 0 & Sx_r & -r_r \end{vmatrix} \begin{vmatrix} i_q \\ i_d \\ i_{qr} \\ i_{dr} \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{vmatrix} \begin{vmatrix} V_d \\ V_q \\ 0 \\ 0 \end{vmatrix} \quad (13.83)$$

where r_s, r_r, x_s, x_r, x_m are the p.u. values of stator (rotor) resistances and stator, rotor and magnetization reactances

$$r_s = \frac{R_s}{X_n}; \quad x_s = \frac{\omega_0 L_s}{X_n}; \quad X_n = \frac{V_{nph}}{I_{nph}} \quad (13.84)$$

$$H = \frac{J \left(\frac{\omega_0}{p_l} \right)^2}{S_n} \text{ (seconds)} \quad (13.85)$$

$$V_d + jV_q = \frac{2}{3} \left(V_a + V_b e^{j\frac{2\pi}{3}} + V_c e^{-j\frac{2\pi}{3}} \right) e^{-j\omega_0 t} \quad (13.86)$$

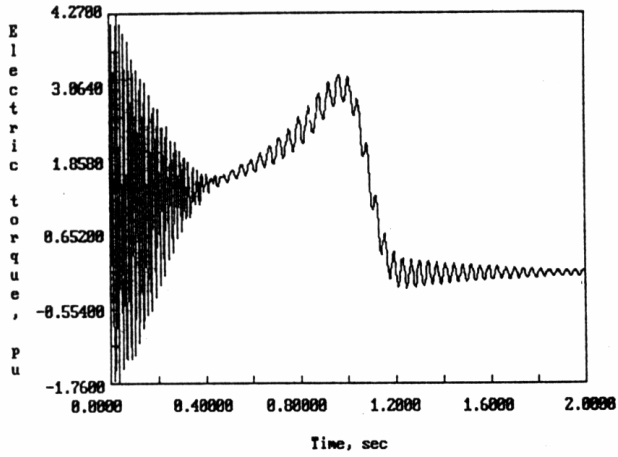
$$\text{The torque is} \quad t_e = x_m (i_{dr} i_q - i_{qr} i_d) \quad (13.87)$$

Equations (13.82) and (13.83) may be solved by numerical methods such as Runge–Kutta–Gill. For the data [15] $r_s = 0.0453$ p.u., $x_s = 2.1195$ p.u., $r_r = 0.0272$ p.u., $x_r = 2.0742$ p.u., $x_m = 2.042$ p.u., $H_M = 0.3$ seconds, $H_L = 0.74$ seconds, $D_L = D_M = 0$, $D_{ML} = 0.002$ p.u./(rad/s), $K_S = 30$ p.u./rad, $T_L = 0.0$, $\omega_0 = 377$ rad/s, the electromagnetic torque (t_e), the shaft torque (t_{sh}), and the speed ω_r are shown in [Figure 13.23](#). [15]

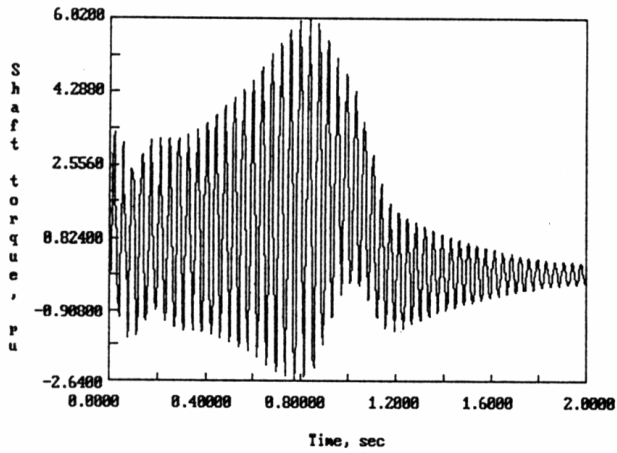
The current transients include a stator frequency current component in the rotor, a slip frequency current in the rotor, and d.c. decaying components both in the stator and rotor. As expected, their interaction produces four torque components: unidirectional torque (by the rotating field components), at supply frequency ω_0 , slip frequency $S \omega_0$, and at speed frequency ω_m .

The three a.c. components may interact with the mechanical part whose natural frequency f_m is [15]

$$f_m = \frac{1}{2\pi} \sqrt{\frac{K_s(H_M + H_L)}{2H_M H_L}} = 26\text{Hz} \quad (13.88)$$



a- Electromagnetic torque



b- Shaft torque

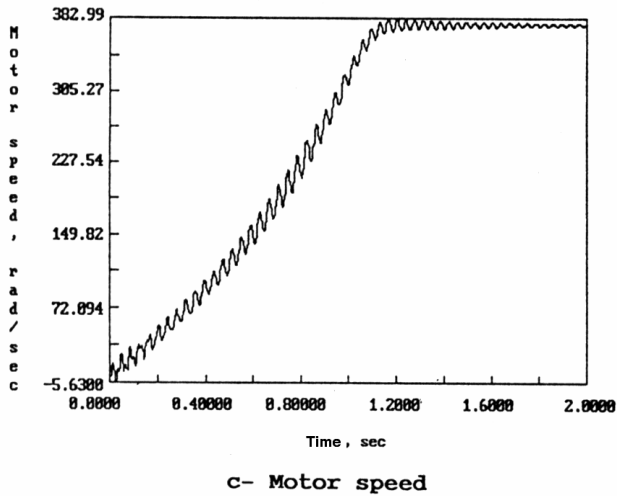


Figure 13.23 Starting transients of a large motor with elastic coupling of inertial load

The torsional torque in [Figure 13.23b](#) (much higher than the electromagnetic torque) may be attributed to the interaction of the slip frequency component of electromagnetic torque, which starts at 60 Hz ($S = 1$) and reaches 26 Hz as the speed increases ($S \cdot 60 \text{ Hz} = 26 \text{ Hz}$, $\omega_m = (1 - S) \cdot \omega_0 = 215 \text{ rad/s}$). The speed frequency component (ω_m) of electromagnetic torque is active when $\omega_m = 162 \text{ rad/s}$ ($\omega_m = 2\pi f_m$), but it turns out to be small.

A good start would imply the avoidance of torsional torques to prevent shaft damage.

For example, in the star/delta connection starting, the switching from star to delta may be delayed until the speed reaches 80% of rated speed to avoid the torsional torques occurring at 215 rad/s and reducing the current and torque peaks below it.

The total starting time with star/delta is larger than for direct (full voltage) starting, but shaft damage is prevented.

Using an autotransformer with 40%, 60%, 75%, 100% voltage steps further reduces the shaft torque but at the price of even larger starting time.

Also, care must be exercised to avoid switching voltage steps near $\omega_m = 215 \text{ rad/s}$ (in our case).

13.10. THE SUDDEN SHORT-CIRCUIT AT TERMINALS

Sudden short-circuit at the terminals of large induction motors produces severe problems in the corresponding local power grid. [16]

Large induction machines with cage rotors are skin-effect influenced so the double cage representation is required. The space-phasor model for double-cage IMs is obtained from (13.66) with $\bar{V}_r = 0$ and eliminating the stator core loss equation, with the rotor core loss equation now representing the second rotor cage. In rotor coordinates,

$$\begin{aligned}\bar{V}_s &= R_s \bar{i}_s + p \bar{\Psi}_s + j \omega_r \bar{\Psi}_s; \quad \bar{\Psi}_s = \bar{\Psi}_m + L_{sl} \bar{i}_s; \quad \bar{\Psi}_m = L_m \bar{i}_m \\ 0 &= R_{r1} \bar{i}_{r1} + p \bar{\Psi}_{r1}; \quad \bar{i}_m = \bar{i}_s + \bar{i}_{r1} + \bar{i}_{r2}; \quad \bar{\Psi}_{r1} = \bar{\Psi}_m + L_{r11} \bar{i}_{r1} \\ 0 &= R_{r2} \bar{i}_{r2} + p \bar{\Psi}_{r2}; \quad \bar{\Psi}_{r2} = \bar{\Psi}_m + L_{r12} \bar{i}_{r2}\end{aligned} \quad (13.89)$$

To simplify the solution, we may consider that the speed remains constant (eventually with initial slip $S_0 = 0$).

Eliminating $\bar{i}_{r1}$ and $\bar{i}_{r2}$ from (13.89) yields

$$\bar{\Psi}_s(p) = L(p) \bar{i}_s(p) \quad (13.90)$$

The operational inductance $L(p)$ is

$$L(p) = L_s \frac{(1 + T'p)(1 + T''p)}{(1 + T_0'p)(1 + T_0''p)} \quad (13.91)$$

The transient and subtransient inductances L_s' , L_s'' are defined as for synchronous machines.

$$\begin{aligned}L_s &= \lim_{s \rightarrow 0} L(p) \\ L_s' &= \lim_{p \rightarrow \infty} (L(p))_{T''=T_0''=0} = L_s \frac{T'}{T_0'} < L_s\end{aligned} \quad (13.92)$$

$$L_s'' = \lim_{p \rightarrow \infty} L(p) = L_s \frac{T' T''}{T_0' T_0''} \ll L_s \quad (13.93)$$

Using Laplace transformation (for $\omega_r = \omega_1 = ct$) makes the solution much easier to obtain. First, the initial (d.c.) stator current at no-load in rotor (synchronous) coordinates ($\omega_r = \omega_1$) $i_s^0(t)$ is

$$i_s^0 = \frac{V_s e^{j\gamma}}{j \omega_1 L_s}; \quad V_s = V \sqrt{2}; R_s \approx 0 \quad (13.94)$$

$$V_{a,b,c} = V \sqrt{2} \cos \left(\omega_1 t + \gamma - (i-1) \frac{2\pi}{3} \right) \quad (13.95)$$

To short-circuit the machine model, $-V_s e^{j\gamma}$ must be applied to the terminals and thus Equation (13.89) with (13.90) yields (in Laplace form):

$$-\frac{V_s e^{j\gamma}}{p} = [R_s + (p + j\omega_1)L(p)]\tilde{i}_s(p) \quad (13.96)$$

Denoting
$$\alpha = \frac{R_s}{L(p)} \approx \frac{R_s}{L_s''} = ct \quad (13.97)$$

the current $i_s(p)$ becomes

$$\tilde{i}_s(s) = \frac{-V_s e^{j\gamma}}{pL(p)(p + j\omega_1 + \alpha)} \quad (13.98)$$

with $L(p)$ of (13.91) written in the form

$$\frac{1}{L(p)} = \frac{1}{L_s} + \frac{\left(\frac{1}{L_s'} - \frac{1}{L_s}\right)p}{\frac{1}{T'} + p} + \frac{\left(\frac{1}{L_s''} - \frac{1}{L_s'}\right)p}{\frac{1}{T''} + p} \quad (13.99)$$

$\tilde{i}_s(p)$ may be put into an easy form to solve.

Finally, $i_s(t)$ is

$$\begin{aligned} \tilde{i}_s(t) = -\frac{V_s}{j\omega_1} e^{j\gamma} & \left[\frac{1}{L_s} \left(1 - e^{-(\alpha + j\omega_1)t}\right) + \left(\frac{1}{L_s'} - \frac{1}{L_s}\right) \left(e^{-\frac{t}{T'}} - e^{-(\alpha + j\omega_1)t} \right) \right. \\ & \left. + \left(\frac{1}{L_s''} - \frac{1}{L_s'}\right) \left(e^{-\frac{t}{T''}} - e^{-(\alpha + j\omega_1)t} \right) \right] \end{aligned} \quad (13.100)$$

Now the current in phase a, $i_a(t)$, is

$$\begin{aligned} i_a(t) &= \text{Re} \left[\tilde{i}_s(t) + \tilde{i}_s^0(t) e^{j\omega_1 t} \right] = \\ V\sqrt{2} & \left[\frac{1}{\omega_1 L_s} e^{-\alpha t} \sin \gamma - \left[\left(\frac{1}{\omega_1 L_s'} - \frac{1}{\omega_1 L_s} \right) e^{-\frac{t}{T'}} + \left(\frac{1}{\omega_1 L_s''} - \frac{1}{\omega_1 L_s'} \right) e^{-\frac{t}{T''}} \right] \sin(\omega_1 t + \gamma) \right] \end{aligned} \quad (13.101)$$

A few remarks are in order.

- The subtransient component, related to α , T'' , L_s'' reflects the decay of the leakage flux towards the rotor surface (upper cage)
- The transient component, related to T' , L_s' , reflects the decay of leakage flux pertaining to the lower cage
- The final value of short-circuit current is zero

The torque expression first requires the stator flux solution.

$$T_e = \frac{3}{2} p_1 (\overline{\Psi}_s \bar{i}_s^*) \quad (13.102)$$

The initial value of stator flux ($R_s \approx 0$) is

$$\overline{\Psi}_s^0 = \frac{V_s e^{j\gamma}}{j\omega_1} = L_s i_s^0 \quad (13.103)$$

with i_s^0 from (13.94).

After short-circuit,

$$\Psi_s(p) = \frac{-V_s e^{j\gamma}}{p[p + j\omega_1 + \alpha]} = L(p) i_s(p) \quad (13.104)$$

with $i_s(p)$ from (13.98).

$$\text{So,} \quad \overline{\Psi}_s(t) = \frac{-V_s e^{j\gamma}}{\alpha + j\omega_1} (1 - e^{-(\alpha + j\omega_1)t}) \quad (13.105)$$

We may neglect α and add $\overline{\Psi}_s(t)$ with $\overline{\Psi}_s^0$ to obtain $\overline{\Psi}_{st}(t)$.

$$\overline{\Psi}_{st}(t) = \frac{V_s e^{j\gamma}}{j\omega_1} e^{-(\alpha + j\omega_1)t} \quad (13.106)$$

Now, from (13.100), (13.105), and (13.102), the torque is

$$T_e = 3V^2 \frac{p_1}{\omega_1} \left[\left(\frac{1}{\omega_1 L_s'} - \frac{1}{\omega_1 L_s} \right) e^{-\frac{t}{T'}} + \left(\frac{1}{\omega_1 L_s''} - \frac{1}{\omega_1 L_s'} \right) e^{-\frac{t}{T''}} \right] \sin(\omega_1 t) \quad (13.107)$$

Neglecting the damping components in (13.107), the peak torque is

$$T_{\text{peak}} \approx -\frac{3p_1 V^2}{\omega_1} \frac{1}{\omega_1 L_s''} \quad (13.108)$$

This result is analogous to that of peak torque at start up.

Typical parameters for a 30 kW motors are [17, 18]

$$T' = 50.70 \text{ ms}, T'' = 3.222 \text{ ms}, T_0' = 590 \text{ ms}, T_0'' = 5.244 \text{ ms}, 1/\alpha = 20.35 \text{ ms}, L_m = 30.65 \text{ mH}, L_{sl} = 1 \text{ mH} (L_s = 31.65 \text{ mH}), R_s = 0.0868 \Omega$$

Analyzing, after acquisition, the sudden short-circuit current, the time constants (T_0' , T_0'' , T' , T'') may be determined by curve-fitting methods.

Using power electronics, the IM may be separated very quickly from the power grid and then short-circuited at various initial voltages and frequency. [17] Such tests may be an alternative to standstill frequency response tests in determining the machine parameters.

Finally, we should note that the torque and current peaks at sudden short-circuit are not (in general) larger than for direct connection to the grid at any

speed. It should be stressed that when direct connection to the grid is applied at various initial speeds, the torque and current peaks are about the same.

Reconnection, after a short turn-off period before the speed and the rotor current have decreased, notably produces larger transients if the residual voltage at stator terminals is in phase opposition with the supply voltage at the reconnection instant.

The question arises: which is the most severe transient? So far apparently there is no definite answer to this question, but the most severe transient up to now is reported in [18].

13.11. MOST SEVERE TRANSIENTS (SO FAR)

We inferred above that reconnection, after a short supply interruption, could produce current and torque peaks higher than for direct starting or sudden short-circuit.

Whatever cause would prolong the existence of d.c. decaying currents in the stator (rotor) at high levels would also produce severe current and torque oscillations due to interaction between the a.c. source frequency current component caused by supply reconnection and the large d.c. decaying currents already in existence before reconnection.

One more reason for severe torque peaks would be the phasing between the residual voltage at machine terminals at the moment of supply reconnection. If the two add up, higher transients occur.

The case may be [13] of an IM supplied through a power transformer whose primary is turned off after starting the motor, at time t_d with the secondary connected to the motor, that is left for an interval Δt , before supply reconnection takes place (Figure 13.24).

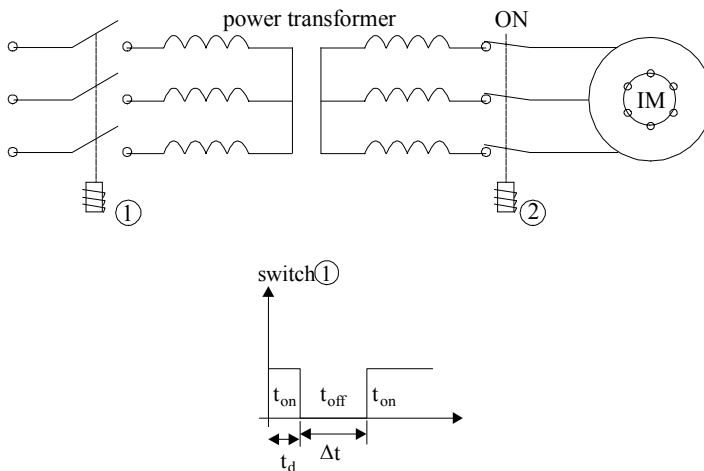


Figure 13.24 Arrangement for most severe transients

During the transformer primary turn off, the total (no-load) impedance of the transformer (r_{ls} , x_{l0s}), as seen from the secondary side, is connected in series with the IM stator resistance and leakage reactance r_s , x_{ls} , given in relative (or absolute) units.

To investigate the above transients, we simply use the d-q model in p.u. in (13.83) with the simple motion equation,

$$H \frac{d\omega_r}{dt} = t_e - t_L \quad (13.109)$$

During the Δt interval, the model is changed only in the sense of replacing r_s with $r_s + r_{ls}$ and x_{ls} with $x_{ls} + x_{l0s}$. When the transformer primary is reconnected, r_{ls} and x_{l0s} are eliminated.

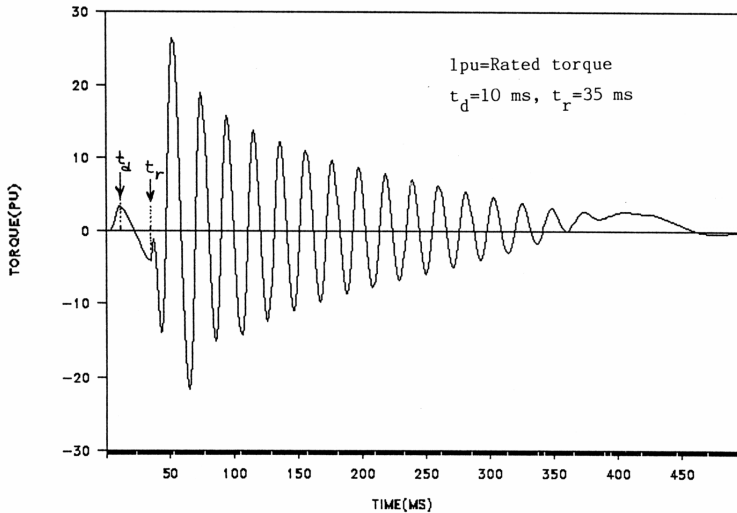
Digital simulations have been run for a 7.5 and a 500 HP IM.

The parameters of the two motors [18] are shown in [Table 13.1](#).

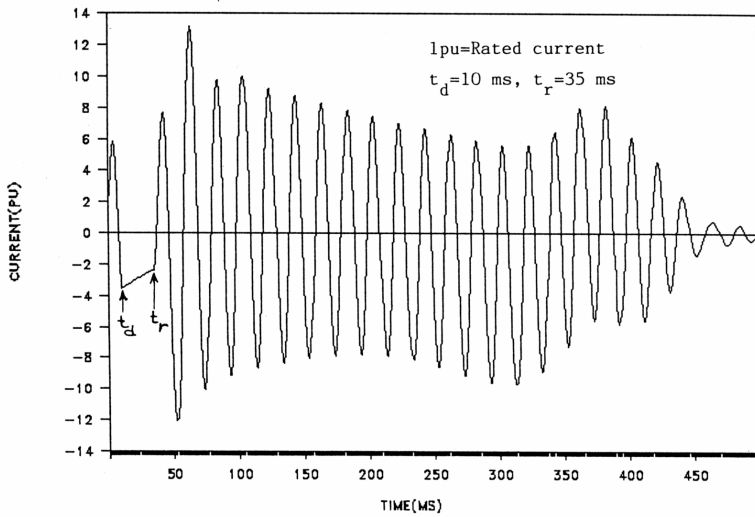
Table 13.1. Machine parameters

Size HP	V_L line	rpm	T_n Nm	I_n A	R_s Ω	X_{ls} Ω	X_m Ω	R_r Ω	X_{lr} Ω	J Kgm ²	f_l Hz
7.5	440	1440	37.2	9.53	0.974	2.463	68.7	1.213	2.463	0.042	50
500	2300	1773	1980	93.6	0.262	1.206	54.04	0.187	1.206	11.06	60

For the 7.5 HP machine, results, as shown in [Figure 13.25a, b, c](#) have been obtained. For the 500 HP machine, [Figure 13.26](#) shows only 12 p.u. peak torque while 26 p.u. peak torque is noticeable for the 7.5 HP machine.

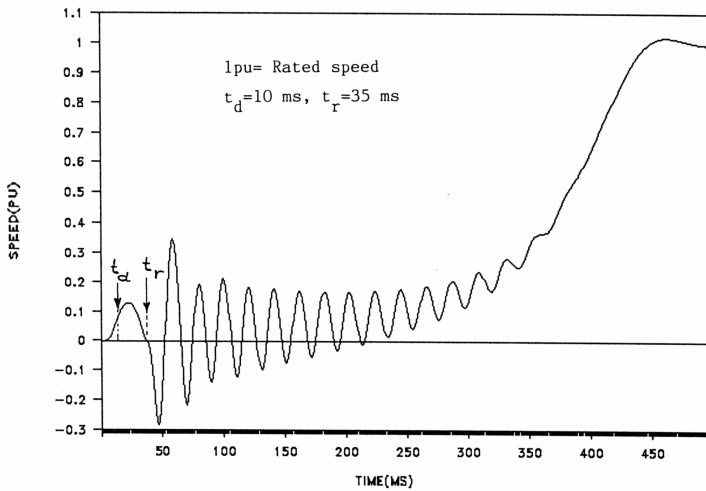


Torque-time pattern-7.5 HP machine



Current-time pattern-7.5 HP machine

b.)



Speed - time pattern - 7.5HP machine

c.)

Figure 13.25 Transformer primary turn-off and reconnection transients (7.5HP)

The influence of turn-off moment t_d (after starting) and of the transformer primary turn-off period Δt on peak torque is shown on [Figure 13.27](#) for the 7.5 HP machines.

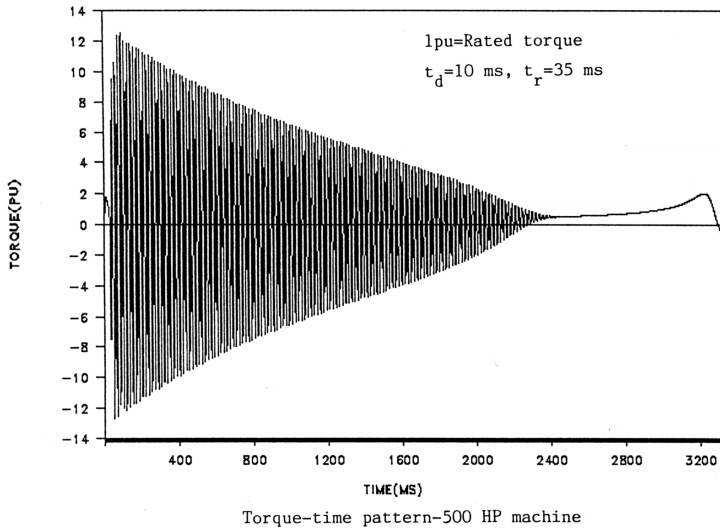


Figure 13.26. Transformer primary turn off and reconnection transients (500HP)

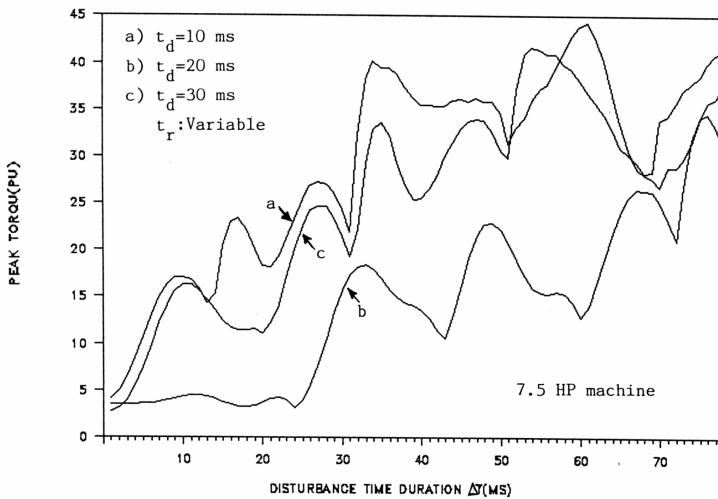


Figure 13.27 Peak torque versus perturbation duration (7.5HP machine)

Figure 13.27 shows very high torque peaks. They depend on the turn-off time after starting t_d . There is a period difference between t_d values for curves a and c and their behavior is similar. This shows that the residual voltage phasing at the reconnection instant is important. If it adds to the supply voltage, the

torque peaks increase. When it subtracts, the peak torques decrease. The periodic behaviour with Δt is a proof of this aspect.

Also, up to a point, when the perturbation time Δt increases, the d.c. (nonperiodic) current components are high as the combined stator and secondary transformer (subtransient) and IM time constant is large.

This explains the larger value of peak torque with Δt increasing. Finally, the speed shows oscillations (even to negative values, for low inertia (power)) and the machine total starting time is notably increased.

Unusually high peak torques and currents are expected with this kind of transients for a rather long time, especially for low power machines while, even for large power IMs, they are at least 20% higher than for sudden short-circuit .

13.12. THE ABC–DQ MODEL FOR PWM INVERTER FED IMs

Time domain modeling of PWM inverter fed IMs, both for steady-state and transients, symmetrical and fault conditions, may be approached through the so called abc–dq model (Figure 13.28).

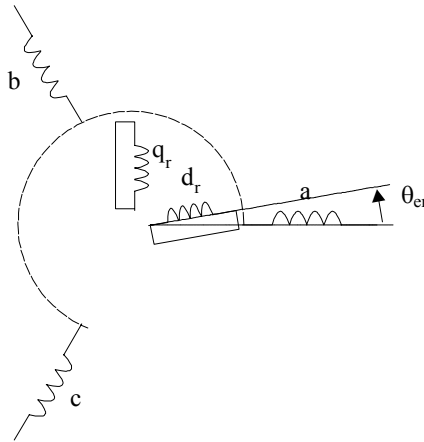


Figure 13.28 The abc – dq model

The derivation of the abc–dq model in stator coordinates may be obtained starting again from, abc–a_rb_rc_r model with Park transformation of only rotor variables:

$$\left| P_{abcdq0} \right| = \begin{bmatrix} [1] & [0] \\ [0] & [P_{3 \times 3}(\theta_b - \theta_{cr})] \end{bmatrix} \quad (13.110)$$

For stator coordinates, $\theta_b = 0$.

Instead of phase voltages, the line voltages may be used:
 V_{ab} , V_{bc} , V_{ca} instead of V_a , V_b , V_c ,

$$V_{ab} = V_a - V_b; \quad V_{bc} = V_b - V_c; \quad V_{ca} = V_c - V_a \quad (13.111)$$

Making use of the 6×6 inductance matrix $\left[L_{abca_r b_r c_r}(\theta_{er}) \right]$ of (13.1), the abc-dq model equations become

$$[V] = [R][I] + \omega_r G[I] + L \frac{d}{dt}[I] \quad (13.112)$$

$$\begin{aligned} [R] &= \text{Diag}[R_s, R_s, R_s, R_r, R_r, R_r] \\ [V] &= [V_{ab}, V_{bc}, V_{ca}, 0, 0] \\ [I] &= [i_a, i_b, i_c, i_{dr}, i_{qr}] \end{aligned} \quad (13.113)$$

$$[G] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{L_m}{\sqrt{2}} & -\frac{L_m}{\sqrt{2}} & 0 & L_r \\ -\sqrt{\frac{2}{3}}L_m & \frac{L_m}{\sqrt{6}} & \frac{L_m}{\sqrt{6}} & -L_r & 0 \end{bmatrix} \quad (13.114)$$

$$[L] = \begin{bmatrix} L_s & -\frac{L_m}{3} & -\frac{L_m}{3} & L_m\sqrt{\frac{2}{3}} & 0 \\ -\frac{L_m}{3} & L_s & -\frac{L_m}{3} & -\frac{L_m}{\sqrt{6}} & \frac{L_m}{\sqrt{2}} \\ -\frac{L_m}{3} & -\frac{L_m}{3} & L_s & -\frac{L_m}{\sqrt{6}} & \frac{L_m}{\sqrt{2}} \\ \sqrt{\frac{2}{3}}L_m & -\frac{L_m}{\sqrt{6}} & -\frac{L_m}{\sqrt{6}} & L_r & 0 \\ 0 & \frac{L_m}{\sqrt{2}} & -\frac{L_m}{\sqrt{2}} & 0 & L_r \end{bmatrix} \quad (13.115)$$

The parameters L_s , L_r , L_m refer to phase inductances,

$$L_s = L_m + L_{ls}; \quad L_r = L_m + L_{lr} \quad (13.116)$$

The torque equation becomes

$$T_e = p_1 [I]^T [G] [I] \quad (13.117)$$

Finally, the motion equation is

$$\frac{J}{p_1} \frac{d\omega_r}{dt} = T_e - T_{load} - B\omega_r \quad (13.118)$$

For PWM excitation and mechanical steady-state ($\omega_r = \text{const.}$) based on $V_{ab}(t)$, $V_{bc}(t)$, $V_{ca}(t)$ functions, as imposed by the corresponding PWM strategy, the currents may be found from (13.112) under the form

$$[\dot{\mathbf{I}}] = -[\mathbf{L}]^{-1}[\mathbf{R}]\mathbf{I} + \omega_r[\mathbf{G}]\mathbf{I} + [\mathbf{L}]^{-1}[\mathbf{V}] \quad (13.119)$$

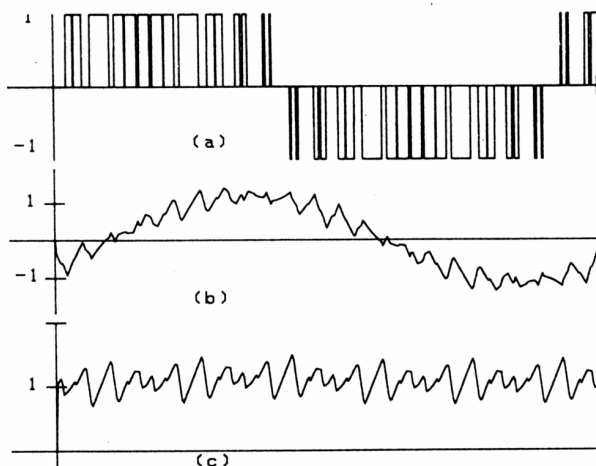


Figure 13.29 Computed waveforms for sinusoidal PWM scheme ($F_{nc} = 21$), $f = 60$ Hz
a.) line to line voltage V_{ab} , b.) line current i_a , c.) developed torque

Initial values of currents are to be given for steady-state sinusoidal supply and the same fundamental.

Although a computer program is required, the computation time is rather low as the coefficients in (13.119) are constant.

Results obtained for an IM with $R_s = 0.2\Omega$, $R_r = 0.3\Omega$, $X_m = 16\Omega$, and $X_r = X_s = 16.55\Omega$, fed with a fundamental frequency $f_n = 60$ Hz, at a switching frequency $f_c = 1260$ Hz and sinusoidal PWM, are shown in [Figure 13.29](#). [19]

When the speed varies, the PWM switching patterns may change and thus produce current and torque transients. A smooth transition is needed for good performance. The abc-dq model, including the motion equation, also serves this purpose (for example, switching from harmonic elimination PWM ($N = 5$) to square wave, [Figure 13.30](#) [19].)

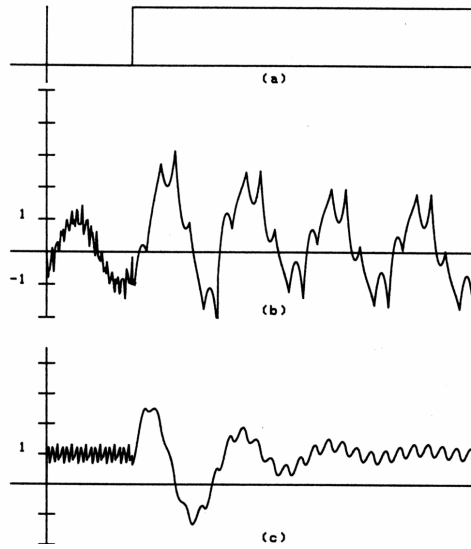


Figure 13.30 Computed transient response for a change from harmonic ($N = 5$) elimination PWM to square wave excitation at 60 Hz
a.) voltage, b.) current, c.) torque

The current waveform changes to the classic six-pulse voltage response after notable current and torque transients.

a.) *Fault conditions*

Open line conditions or single phasing due to loss of gating signals of a pair of inverter switches are typical faults. Let us consider delta connection when line a is opened ([Figure 13.31a](#)). The relationships between the system currents $[i_{ab}, i_{bc}, i_{ca}, i_d, i_q]$ and the “fault” currents $i_{bac}, i_{bc}, i_d, i_q$ are ([Figure 13.31a](#)).

$$\begin{bmatrix} i_{ab} \\ i_{bc} \\ i_{ca} \\ i_d \\ i_q \end{bmatrix} = [C] \begin{bmatrix} i_{bac} \\ i_{bc} \\ i_d \\ i_q \end{bmatrix}; [I_n] = [C][i_0] \quad (13.120)$$

$$[C] = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (13.121)$$

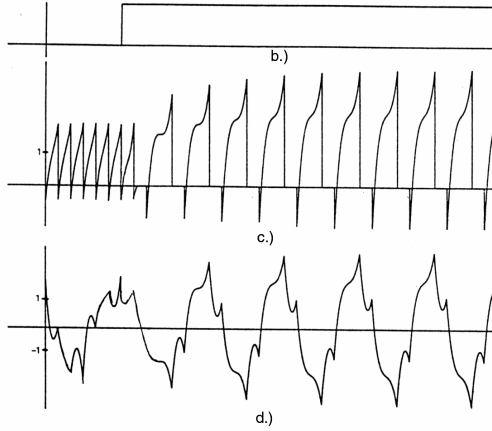
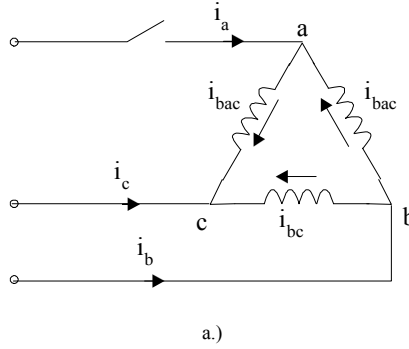


Figure 13.31 Computed transient response for a single phasing fault on line 'a' at 60 Hz
a.) open line condition b.) instant when single phasing occurs
c.) dc link current d.) line current i_s

The voltage relationship is straightforward as power has to be conserved:

$$\begin{bmatrix} V_{bac} \\ V_{bc} \\ V_d \\ V_q \end{bmatrix} = [C]^T \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \\ V_d \\ V_q \end{bmatrix}; \quad [V_n] = [C]^T [V_0] \quad (13.122)$$

Now we may replace the voltage matrix in (13.112) to obtain

$$[V_n] = [C]^T [R][C][I_n] + [C]^T \omega_r [G][C][I_n] + [C]^T [L][C] \frac{d[I_n]}{dt} \quad (13.123)$$

All we need to be able to solve Equation (13.123) are the initial conditions.

In reality, (Figure 13.31a) at time $t = 0$, the currents i_{ba} and i_{ca} are forced to be equal to I_{bac} when open line occurs. To avoid a discontinuity in the stored energy, the flux conservation law is applied to (13.112).

We impose $i_{ab} = i_{ca} = -i_{bac}$ and the conservation of flux will give the variation of rotor current to comply with flux conservation. Typical results for such a transient [19] are shown in Figure 13.31b,c,d. Severe d.c. line pulsations and line b current transients are visible.

Other fault conditions may be treated in a similar way.

13.13. FIRST ORDER MODELS OF IMs FOR STEADY-STATE STABILITY IN POWER SYSTEMS

The quasistatic (first order) or slip model of IMs with neglected stator and rotor transients may be obtained from the general model (13.70) with

$$\frac{d\bar{\Psi}_s}{dt} = \frac{d\bar{\Psi}_r}{dt} = 0 \text{ in synchronous coordinates.}$$

$$\begin{aligned} (1 + j\omega_l \tau_s) \bar{\Psi}_s &= \tau_s V_s + K_r \bar{\Psi}_r \\ (1 + j(\omega_l - \omega_r)) \bar{\Psi}_r &= K_s \bar{\Psi}_s \\ T_e &= -\frac{3}{2} p_1 \operatorname{Re} \left(j \bar{\Psi}_r \bar{i}_r^* \right), \quad \bar{i}_r = \sigma^{-1} \left(\frac{\bar{\Psi}_r}{L_r} - \bar{\Psi}_s \frac{L_m}{L_s L_r} \right) \\ \frac{J}{p_1} \frac{d\omega_r}{dt} &= T_e - T_{\text{load}} \end{aligned} \quad (13.124)$$

It has been noted that this overly simplified speed model becomes erroneous for large induction machines.

Near rated conditions, the structural dynamics of low and large power induction machines differ. The dominant behavior of low power IMs is a first order speed model while the large power IMs is characterized by a first order voltage model. [20, 21]

Starting again with the third order model of (13.70), we may write the rotor flux space phasor as

$$\bar{\Psi}_r = \Psi_r e^{j\delta} \quad (13.125)$$

This way we may separate the real and imaginary part of rotor equation of (13.70) as

$$\begin{aligned} \frac{\tau_r}{K_r} \frac{d\Psi_r}{dt} K_r &= \frac{x}{x'} \Psi_r K_r + \frac{x - x'}{x'} V \cos \delta; \quad K_r \approx K_s = \frac{L_m}{L_s} = \frac{L_m}{L_r} \\ \frac{d\delta}{dt} &= \omega_r - \omega_l - \frac{x - x'}{x'} K_r \frac{V \sin \delta}{\tau_r \Psi_r K_r} \end{aligned}$$

$$H' \frac{d\omega_r}{dt} = -V \frac{\Psi_r}{x'} K_r \sin \delta - t_{\text{load}} \quad (13.125')$$

where V (voltage), Ψ_r – rotor flux amplitude, x – no-load reactance, x' – short-circuit (transient) reactance in relative units (p.u.) and τ_r' , ω_1 , ω_r in absolute units and H' in seconds². The stator resistance is neglected.

In relative units, $\Psi_r K_r$ may be replaced by the concept of the voltage behind the transient reactance x' , E' .

$$\begin{aligned} \frac{\tau_r'}{K_r} \frac{dE'}{dt} &= \frac{x}{x'} E' + \frac{x-x'}{x'} V \cos \delta \\ \frac{d\delta}{dt} &= \omega_r - \omega_1 - \frac{x-x'}{x'} K_r \frac{V \sin \delta}{\tau_r' E'} \\ H' \frac{d\omega_r}{dt} &= -V \frac{E'}{x'} \sin \delta - t_{\text{load}} \end{aligned} \quad (13.126)$$

- The concept of E' is derived from the similar synchronous motor concept
- The angle δ is the rotor flux vector angle with respect to the synchronous reference system
- The steady-state torque is obtained from (13.126) with $d/dt = 0$

$$E' = \frac{x-x'}{x} V \cos \delta \quad (13.127)$$

$$\omega_r - \omega_1 = \frac{x-x'}{x'} K_r \frac{V \sin \delta}{\tau_r' E'} < 0 \text{ for motoring} \quad (13.128)$$

$$t_e = -V \frac{E'}{x'} \sin \delta = -\left(\frac{1}{x'} - \frac{1}{x}\right) \frac{V^2}{2} \sin 2\delta > 0 \text{ for motoring} \quad (13.129)$$

for positive torque $\delta < 0$ and for peak torque $\delta_K = \pi/4$ as in a fictitious reluctance synchronous motor with $x_d = x$ (no-load reactance) and $x_q = x'$ (transient reactance).

For low power IMs, E' and δ are the fast variables and $\omega_r - \omega_1$, the slow variable.

Consequently, we may replace δ and E' from the steady-state Equations (13.127) and (13.128) in the motion equation (13.126).

$$x' H' \frac{d(\omega_r - \omega_1)}{dt} = -V^2 \frac{x-x'}{x} \frac{(\omega_r - \omega_1) T'}{1 + T'(\omega_r - \omega_1)^2} - x' t_{\text{load}}; \quad T' = \frac{\tau_r'}{K_r} \quad (13.130)$$

Also, from (13.127) and (13.128),

$$E' = \frac{(x - x')V}{x\sqrt{1 + T'^2(\omega_r - \omega_1)^2}}; \delta \approx \tan^{-1}T'(\omega_r - \omega_1) \quad (13.131)$$

In contrast, for large power induction machines, the slow variable is E' and the fast variables are δ and $\omega_r - \omega_1$. With $d\omega_r/dt = 0$ and $d\delta/dt = 0$ in (13.126),

$$\delta = \sin^{-1}\left(\frac{-x't_{\text{load}}}{VE'}\right); (\omega_r - \omega_1) = 0 \quad (13.132)$$

It is evident that $\omega_r = \omega_1 \neq 0$.

A nontrivial first order approximation of $\omega_r - \omega_1$ is [21]

$$\omega_r - \omega_1 \approx \frac{x't_{\text{load}}}{T'\sqrt{(VE')^2 - (x't_{\text{load}})^2}} \quad (13.133)$$

Substituting δ of (13.132) in (13.126) yields

$$T' \frac{dE'}{dt} = -\frac{x}{x'}E' + \frac{x - x'}{x'}V\sqrt{1 - \left(\frac{x't_{\text{load}}}{VE'}\right)^2} \quad (13.134)$$

Equations (13.132) through (13.134) represent the modified first order model for large power induction machines. Good results with these first order models are reported in [20] for 50 HP and 500 HP, respectively, in comparison with the third order model (Figure 13.32).

Still better results for large IMs are claimed in [21] with a heuristic first order voltage model based on sensitivity studies.

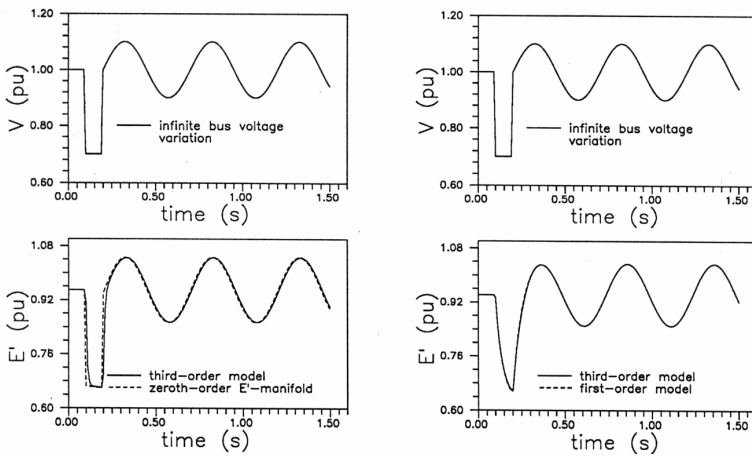


Figure 13.32 Third and first order model transients a.) 50 HP

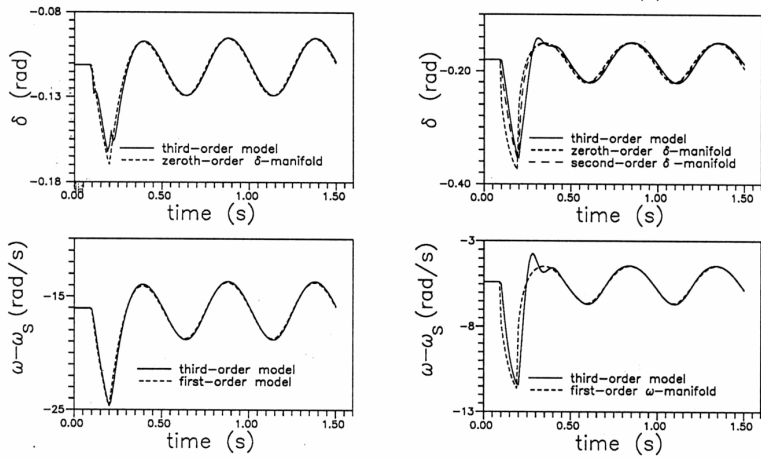


Figure 13.32 (continued) Third and first order model transients b.) 500 HP, IM

13.14. MULTIMACHINE TRANSIENTS

A good part of industrial loads is represented by induction motors. The power range of induction motors in industry spans from a few kW to more than 10 MW per unit. A local (industrial) transformer feeds a group of induction motors through pertinent switchgears which may undergo randomly load perturbations, direct turn on starting or turn off. Alternatively, bus transfer schemes are providing energy-critical duty loads to thermal nuclear power plants, etc.

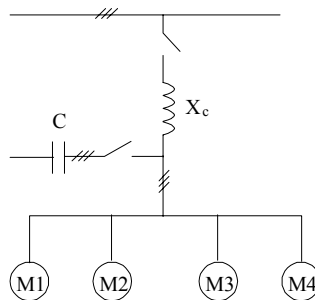


Figure 13.33 A group of induction motors

During the time interval between turn off from one power grid until the turn on to the emergency power grid, the group of induction motors, with a series reactance common feeder or with a parallel capacitor bank (used for power factor correction), exhibits a residual voltage, which slowly dies down until reconnection takes place (Figure 13.33). During this period, the residual rotor

currents in the various IMs of the group are producing stator emfs and, depending on their relative phasing and amplitude (the speed or inertia), some of them will act as generators and some as motors until their mechanical and magnetic energies dry out.

The obvious choice to deal with multimachine transients is to use the complete space phasor model with flux linkages $\bar{\Psi}_s$ and $\bar{\Psi}_r$ and ω_r as variables ((13.54 through 13.56) and (13.64)).

Adding up to the 5th order model of all n IMs (5n equations), the equations related to the terminal capacitor (and the initial conditions before the transient is initiated) seems the natural way to solve any multimachine transient. A unique synchronous reference system may be chosen to speed up the solution.

Furthermore, to set the initial conditions, say, when the group is turned off, we may write that total stator current $\bar{i}_{st}(0+) = 0$. With a capacitor at the terminal, the total current is zero.

$$\sum_{j=1}^n \bar{i}_{sj}(0+) + \bar{i}_c = 0; \quad \frac{d\bar{V}_s}{dt} = \frac{\bar{i}_c}{C} \quad (13.135)$$

with $\bar{i}_s$ from (13.54),

$$\sum_{j=1}^n \bar{i}_{sj} = \sum_{j=1}^n \sigma_j^{-1} \left(\frac{\bar{\Psi}_{sj}}{L_{sj}} - \frac{\bar{\Psi}_{rj} L_{mj}}{L_{sj} L_{rj}} \right) \quad (13.136)$$

Differentiating Equations (13.135) with (13.136) with respect to time will produce an equation containing a relationship between the stator and rotor flux time derivatives. This way, the flux conservation law is met.

We take the expressions of these derivatives from the space phasor model of each machine and find the only unknown, $(\bar{V}_s)_{t=0+}$, as the values of all variables $\bar{\Psi}_{sj}$, $\bar{\Psi}_{rj}$, ω_{rj} , before the transient is initiated, are known.

Now, if we neglect the stator transients and make use of the third order model, we may use the stator equations of any machine in the group to calculate the residual voltage $\bar{V}_s(t)$ as long as (13.135) with (13.136) is fulfilled.

This way no iteration is required and only solving each third model of each motor is, in fact, done. [22, 23] Unfortunately, although the third order model forecasts the residual voltage $V_s(t)$ almost as well as the full (5th order) model, the test results show a sudden drop in this voltage in time. This sharply contrasts with the theoretical results.

A possible explanation for this is the influence of magnetic saturation which maintains the “selfexcitation” process of some of the machines with larger inertia for some time, after which a fast deexcitation of them by the others, which act as motors, occurs.

Apparently magnetic saturation has to be accounted for to get any meaningful results related to residual voltage transient with or without terminal capacitors.

Simpler first order models have been introduced earlier in this chapter. They have been used to study load variation transients once the flux in the machine is close to its rated value. [24, 25]

Furthermore, multimachine transients (such as limited time bus fault, etc.) with unbalanced power grid voltages have also been treated, based on the dq (space phasor) model—complete or reduced. However, in all these cases, no strong experimental validation has been brought up so far.

So we feel that model reduction, or even saturation and skin effect,, has to be pursued with extreme care when various multimachine transients are investigated.

13.15. SUBSYNCHRONOUS RESONANCE (SSR)

Subsynchronous series resonance may occur when the induction motors are fed from a power source that shows a series capacitance (Figure 13.34). It may lead to severe oscillations in speed, torque, and current. It appears, in general, during machine starting or after a power supply fault.

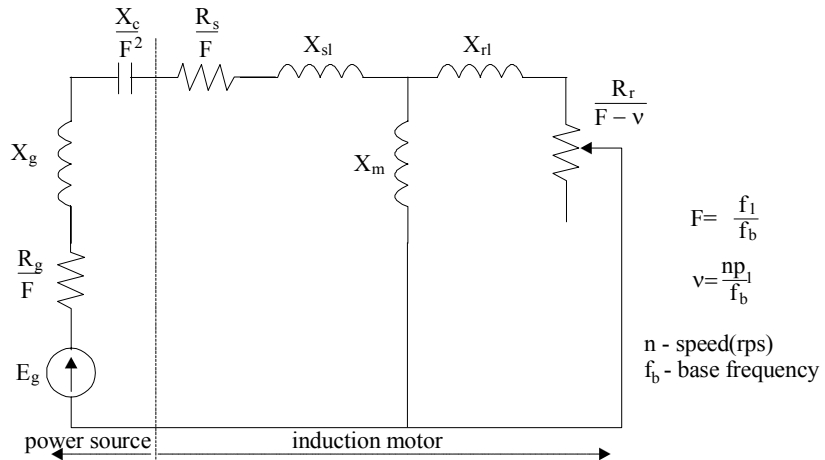


Figure 13.34 Equivalent circuit with power source parameters, R_g , X_g , X_c in series

f_b is the base frequency, f_l is the current frequency, n is the rotor speed. The reactances are written at base frequency.

It is self evident that the space phasor model may be used directly to solve this problem completely, provided a new variable (capacitor voltage V_c) is introduced.

$$\frac{d\bar{V}_c}{dt} = \frac{1}{C} \bar{i}_s \quad (13.137)$$

The total solution contains a forced solution and a free solution. The complete mathematical model is complex when magnetic saturation is considered. It simplifies greatly if the magnetization reactance X_m is considered constant (though at a saturated value). In contrast to the induction capacitor-excited generator, the free solution here produces conditions that lead to machine heavy saturation.

In such conditions we may treat separately the resonance conditions for the free solution.

To do so we lump the power source impedance R_g, X_g into stator parameters to get

$$R_e = R_s + R_g; \quad X_e = X_g + X_{sl} \quad (13.138)$$

and eliminate the generator (power source) emf E_g (Figure 13.35).

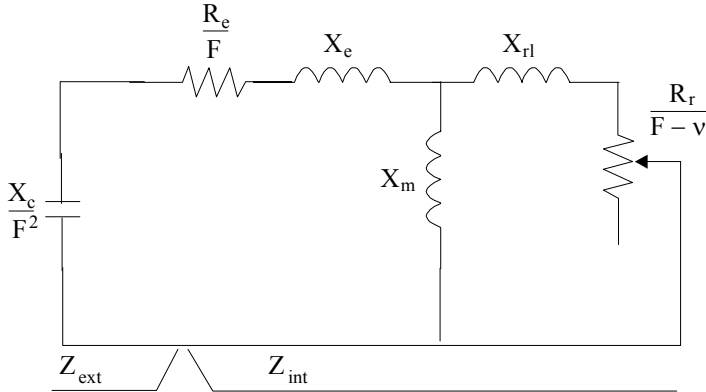


Figure 13.35 Equivalent circuit for free solution

To simplify the analysis, let us use a graphical solution. The internal impedance locus is, as known, a circle with the center 0 at $0 \left(\frac{R_e}{F}, a \right)$. [28]

$$a \approx X_e + \frac{X_e^2}{2(X_e + X_m)} \quad (13.139)$$

and the radius r

$$r \approx \frac{X_e^2}{2(X_e + X_m)} \quad (13.140)$$

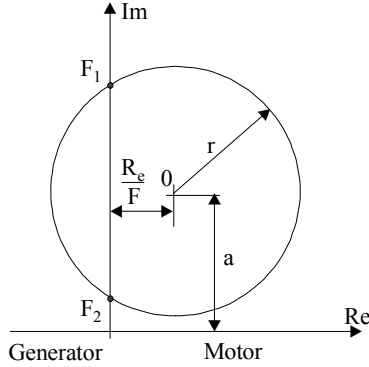


Figure 13.36 Impedance locus

On the other hand, the external (the capacitor, in fact) impedance is represented by the imaginary axis (Figure 13.36). The intersection of the circle with the imaginary axis, for a given capacitor, takes place in two points F_1 and F_2 which correspond to reference conditions. F_1 corresponds to the lower slip and is the usual self excitation point of induction generators.

The p.u. values of frequencies F_1 and F_2 , for a given series capacitor X_c and resistor, R_e , reactance X_e , are

$$F_1 \approx \sqrt{\frac{X_e + X_m}{X_c}} \quad (13.141)$$

$$F_2 \approx \sqrt{\frac{X_e + \frac{X_m X_{rl}}{X_m + X_{rl}}}{X_c}}$$

Alternatively, for a given resonance frequency and capacitor, we may obtain the resistance R_e to cause subsynchronous resonance. With $F = F_1$ given, we may use the impedance locus to obtain R_e .

$$\frac{X_c}{F_1^2} = a + \sqrt{r^2 - \left(\frac{R_e}{F_1}\right)^2} \quad (13.142)$$

Also, from the internal impedance at $R_e(Z_{int}) = 0$ equal to X_c/F_1^2 , the value of speed (slip: $F-v$) may be obtained.

$$\text{slip} = F_1 - v \approx \pm \sqrt{\frac{R_r \left(X_m - X_c - \frac{X_c}{F_1^2} \right)}{(X_m + X_{rl}) \left[\left(\frac{X_c}{F_1^2} - X_e \right) (X_m + X_{rl}) - X_m X_{rl} \right]}} \quad (13.143)$$

In general, we may use Equation (13.143) with both $\pm$ to calculate the frequencies for which resonance occurs.

$$F^4(a^2 - r^2) + F^2(R_e^2 - 2aX_c) + X_c^2 = 0 \quad (13.144)$$

The maximum value of R_e , which still produces resonance, is obtained when the discriminant of (13.144) becomes zero.

$$R_{e\max} = \sqrt{2X_c(\sqrt{a^2 - r^2} + a)} = KX_c \quad (13.145)$$

The corresponding frequency,

$$F(R_{e\max}) = \sqrt{\frac{2X_c^2}{2aX_c - R_{e\max}^2}} \quad (13.146)$$

This way the range of possible source impedance that may produce subsynchronous resonance (SSR) has been defined.

Preventing SSR may be done either by increasing R_e above $R_{e\max}$ (which means notable losses) or by connecting a resistor R_{ad} in parallel with the capacitor (Figure 13.37), which means less losses.

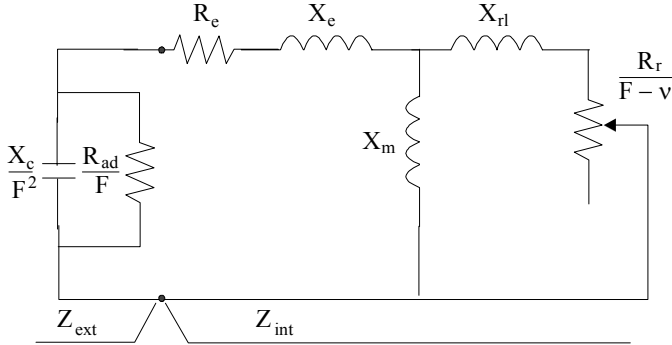


Figure 13.37 Additional resistance R_{ad} for preventing SSR

A graphical solution is at hand as Z_{int} locus is already known (the circle). The impedance locus of the $R_{ad}||X_c$ circuit is a half circle with the diameter R_{ad}/F and the center along the horizontal negative axis (Figure 13.38).

The SSR is eliminated if the two circles, at same frequency, do not intersect each other. Analytically, it is possible to solve the equation:

$$Z_{ext} + Z_{int} = 0 \quad (13.146')$$

again, with $R_{ad}||X_c$ as Z_{ext} .

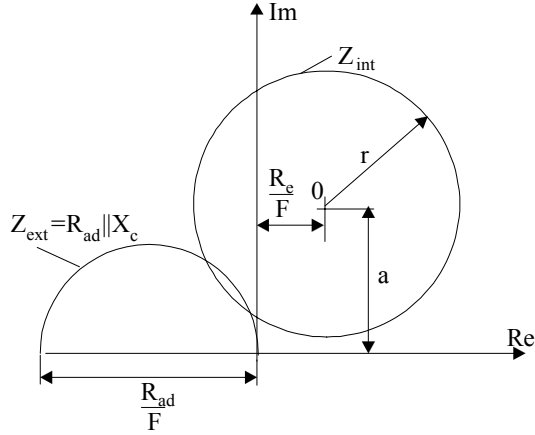


Figure 13.38 Impedances locus for SSR prevention

A 6th order polynomial equation is obtained. The situation with a double real root and 4 complex roots corresponds to the two circles being tangent. Only one SSR frequency occurs. This case defines the critical value of R_{ad} for SSR.

In general, R_{ad} (critical) increases with X_c and so does the corresponding SSR frequency. [27, 28]

Using a series capacitor for starting medium power IMs to reduce temporary supply voltage sags and starting time is a typical situation when the prevention SSR is necessary.

13.16. THE m/N_r ACTUAL WINDING MODELING FOR TRANSIENTS

By m/N_r winding model, we mean the induction motor with m stator windings and N_r actual (rotor loop) windings. [29]

In the general case, the stator windings may exhibit faults such as local short-circuits, or some open coils, when the self and mutual inductances of stator windings have special expressions that may be defined using the winding function method. [30, 31, 32]

While such a complex methodology may be justifiable in large machine preventive fault diagnostics, for a symmetrical stator the definition of inductances is simpler.

The rotor cage is modeled through all its N_r loops (bars). One more equation for the end ring is added. So the machine model is characterized by $(m + N_r + 1 + 1)$ equations (the last one is the motion equation). Rotor bar and end ring faults so common in IMs may thus be modeled through the m/N_r actual winding model.

The trouble is that it is not easy to measure all inductances and resistances entering the model. In fact, relying on analysis (or field) computation is the preferred choice.

The stator phase equations (in stator coordinates) are

$$\begin{aligned}
V_a &= R_s i_a + \frac{d\Psi_a}{dt} \\
V_b &= R_s i_b + \frac{d\Psi_b}{dt} \\
V_c &= R_s i_c + \frac{d\Psi_c}{dt}
\end{aligned} \tag{13.147}$$

The rotor cage structure and unknowns are shown in [Figure 13.39](#).

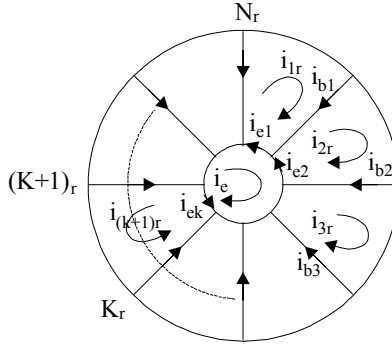


Figure 13.39 Rotor cage with rotor loop currents

The K_r^{th} rotor loop equation (in rotor coordinates) is

$$0 = 2 \left(R_b + \frac{R_e}{N_r} \right) i_{kr} + \frac{d\Psi_{kr}}{dt} - R_b (i_{(k-1)r} + i_{(k+1)r}) \tag{13.148}$$

Also, for one of the end rings,

$$0 = R_e i_e + L_e \frac{di_e}{dt} - \sum_l^{N_r} \left(\frac{R_e}{N_r} i_{kr} + L_e \frac{di_{kr}}{dt} \right) \tag{13.149}$$

As expected, for a healthy cage, the end ring loop cage current $i_e = 0$.

The relationships between the bar loop and end ring currents i_b , i_e are given by Kirchoff's law,

$$i_{bk} = i_{kr} - i_{(k+1)r}; \quad i_{ek} = i_{kr} - i_e \tag{13.150}$$

This explains why only $N_r + 1$ independent rotor current variables remain.

The self and mutual stator phase inductances are given through their standard expressions (see Chapter 5).

$$\begin{aligned}
L_{aa} &= L_{bb} = L_{cc} = L_{ls} + L_{ms} \\
L_{ab} &= L_{bc} = L_{ca} = -\frac{1}{2}L_{ms} \\
L_{ms} &= \frac{4\mu_0(W_l K_{wl})^2 \tau L}{\pi^2 K_c p_l g(1 + K_s)}
\end{aligned} \tag{13.151}$$

The saturation coefficient K_s is considered constant. L_{ls} – leakage inductance, τ – pole pitch, L – stack length, p_l – pole pairs, g – airgap, W_l – turns/phase, K_{wl} – winding factor. The self inductance of the rotor loop calculated on the base of rotor loop area,

$$L_{K_r, K_r} = \frac{2\mu_0(N_r - 1)p_l \tau L}{N_r^2 K_c g(1 + K_s)} \tag{13.152}$$

The mutual inductance between rotor loops is

$$L_{K_r, (K+j)_r} = -\frac{2\mu_0 p_l \tau L}{N_r^2 K_c g(1 + K_s)} \tag{13.153}$$

This is a kind of average small value, as in reality, the coupling between various rotor loops varies notably with the distance between them.

The stator–rotor loops mutual inductances L_{aK_r} , L_{bK_r} , L_{cK_r} are

$$\begin{aligned}
L_{aK_r}(\theta_r) &= L_{sr} \cos[\theta_{er} + (K_r - 1)\alpha] \\
L_{bK_r}(\theta_r) &= L_{sr} \cos\left[\theta_{er} + (K_r - 1)\alpha - \frac{2\pi}{3}\right] \\
L_{cK_r}(\theta_r) &= L_{sr} \cos\left[\theta_{er} + (K_r - 1)\alpha + \frac{2\pi}{3}\right]
\end{aligned} \tag{13.154}$$

$$\alpha = p_l \frac{2\pi}{N_r}; \quad \theta_{er} = p_l \theta_r; \quad p_l - \text{pole pairs} \tag{13.155}$$

$$L_{sr} = \frac{-(W_l K_{wl})\mu_0 2P_l \tau L}{4P_l g K_c (1 + K_s)} \sin \frac{\alpha}{2} \tag{13.156}$$

The $m + N_r + 1$ electrical equations may be written in matrix form as

$$[V] = [R][i] + [L'(\theta_{er})] \left[\frac{di}{dt} \right] + [G][i] \tag{13.157}$$

$$\begin{aligned}
[V] &= [V_a, V_b, V_c, 0, 0, \dots, 0]^T \\
[i] &= [i_a, i_b, i_c, i_{1r}, i_{2r}, \dots, i_{N_r}, i_e]^T
\end{aligned} \tag{13.158}$$

$$L'(\theta_{er}) = \begin{bmatrix} L_{aa}(\theta_{er}) & L_{ab}(\theta_{er}) & & & & & L_{aN_r}(\theta_{er}) & 0 \\ L_{ab}(\theta_{er}) & L_{bb}(\theta_{er}) & & & & & L_{bN_r}(\theta_{er}) & 0 \\ L_{ac}(\theta_{er}) & L_{bc}(\theta_{er}) & & & & & L_{cN_r}(\theta_{er}) & 0 \\ \dots & \dots & & & & & \dots & -\frac{L_e}{N_r} \\ \dots & \dots & & & & & \dots & \dots \\ L_{a3r}(\theta_{er}) & L_{b3r}(\theta_{er}) & & & & & L_{3rN_r}(\theta_{er}) & -\frac{L_e}{N_r} \\ \dots & \dots & & & & & \dots & \dots \\ L_{aN_r}(\theta_{er}) & L_{bN_r}(\theta_{er}) & & & & & L_{N_rN_r}(\theta_{er}) & -\frac{L_e}{N_r} \\ 0 & 0 & 0 & -\frac{L_e}{N_r} & -\frac{L_e}{N_r} & -\frac{L_e}{N_r} & -\frac{L_e}{N_r} & -\frac{L_e}{N_r} + L_e \end{bmatrix} \quad (13.159)$$

$$[G] = \begin{bmatrix} a & b & c & 1r & & & \frac{N_r}{d\theta_{er}} \frac{dL_{aN_r}(\theta_{er})}{d\theta_{er}} & 0 \\ a & \frac{dL_{aa}(\theta_{er})}{d\theta_{er}} & & & & & \frac{dL_{aN_r}(\theta_{er})}{d\theta_{er}} & 0 \\ b & & & & & & & 0 \\ c & & & & & & & 0 \\ 1r & \dots & & & & & \dots & 0 \\ 2r & \dots & & & & & \dots & 0 \\ 3r & & & & & & & 0 \\ \dots & \dots & & & & & \dots & \dots \\ N_r & \frac{dL_{aN_r}(\theta_{er})}{d\theta_{er}} & & & & & \frac{dL_{N_rN_r}(\theta_{er})}{d\theta_{er}} & 0 \\ & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (13.160, 161)$$

$$[R] = \begin{bmatrix} R_s & 0 & 0 & 0 & & & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 & & & & & 0 \\ 0 & 0 & R_s & 0 & & & & & 0 \\ 0 & 0 & 0 & 2\left(R_b + \frac{R_e}{N_r}\right) & -R_b & 0 & 0 & -R_b & -\frac{R_e}{N_r} \\ 0 & 0 & 0 & -R_b & 2\left(R_b + \frac{R_e}{N_r}\right) & -R_b & & 0 & -\frac{R_e}{N_r} \\ \dots & \dots & \dots & 0 & -R_b & \dots & \dots & & -\frac{R_e}{N_r} \\ \dots & \dots & \dots & \dots & & & & -R_b & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & -R_b & 2\left(R_b + \frac{R_e}{N_r}\right) & -\frac{R_e}{N_r} \\ 0 & 0 & 0 & -\frac{R_e}{N_r} & -\frac{R_e}{N_r} & -\frac{R_e}{N_r} & -\frac{R_e}{N_r} & -\frac{R_e}{N_r} & R_e \end{bmatrix}$$

The motion equations are

$$\frac{d\omega_r}{dt} = \frac{p_1}{J} (T_e - T_{load}), \quad \frac{d\theta_{er}}{dt} = \omega_r \quad (13.162)$$

The torque T_e is

$$T_e = p_1 [I]^T \frac{\partial [L(\theta_{er})]}{\partial \theta_{er}} [I] \quad (13.163)$$

Finally,

$$T_e = p_1 L_{sr} \left[\left(i_a - \frac{1}{2} i_b - \frac{1}{2} i_c \right) \sum_1^{N_r} i_{Kr} \sin[\theta_{er} + (K_r - 1)\alpha] + \right. \\ \left. + \frac{\sqrt{3}}{2} (i_b - i_c) \sum_1^{N_r} i_{Kr} \cos[\theta_{er} + (K_r - 1)\alpha] \right] \quad (13.164)$$

A few remarks are in order.

- The $(3 + N_r + 1)$ order system has quite a few coefficients (inductances) depending on rotor position θ_{er} .
- When solving such a system a matrix inversion is required for each integration step when using a Runge–Kutta–Gill or the like method to solve it.
- The case of healthy cage can be handled by a generalized space vector (phasor) approach when the coefficients dependence on rotor position is eliminated.
- The case of faulty bars is handled by increasing the resistance of that (those) bar (R_b) a few orders of magnitude.
- When an end ring segment is broken, the corresponding R_e/N_r term in the resistance matrix is increased a few orders of magnitude (both along the diagonal and along its vertical and horizontal direction in the last row and column, respectively).

In Reference [33], a motor with the following data was investigated:

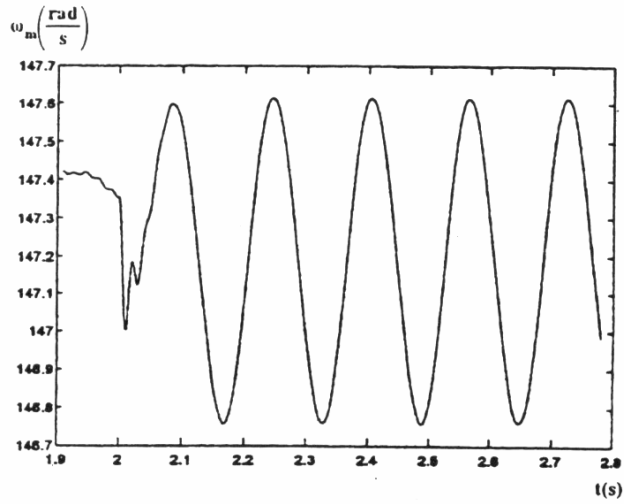
$R_s = 10 \, \Omega$, $R_b = R_e = 155 \, \mu\Omega$, $L_{ls} = 0.035 \, \text{mH}$, $L_{ms} = 378 \, \text{mH}$, $W_1 = 340$ turns/phase, $K_{w1} = 1$, $N_r = 30$ rotor slots, $p_1 = 2$ pole pairs, $L_e = L_b = 0.1 \, \mu\text{H}$, $L_{sr} = 0.873 \, \text{mH}$, $\tau = 62.8 \, \text{mm}$ (pole pitch), $L = 66 \, \text{mm}$ (stack length), $g = 0.375 \, \text{mm}$, $K_c = 1$, $K_s = 0$, $J = 5.4 \cdot 10^{-3} \, \text{Kg m}^2$, $f = 50 \, \text{Hz}$, $V_L = 380 \, \text{V}$, $P_n = 736 \, \text{W}$, $I_0 = 2.1 \, \text{A}$.

Numerical results for bar 2 broken ($R_{b2} = 200R_b$) with the machine under load $T_L = 3.5 \, \text{Nm}$ are shown in (Figure 13.40).

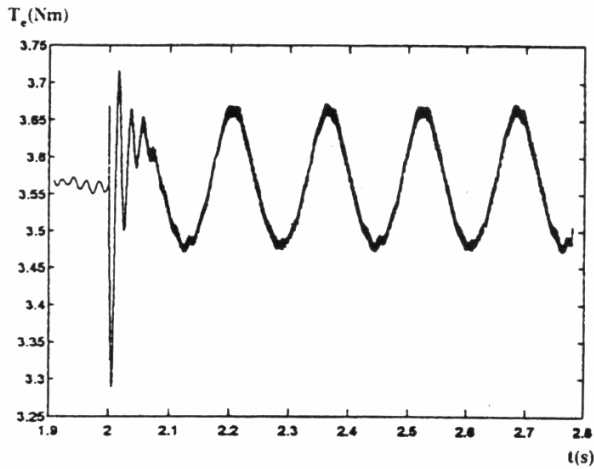
For the ring segment 3 broken ($R_{e3} = 10^3 R_e/N_r$), the speed and torque are given in Figure 13.41 [33].

- Small pulsations in speed and torque are caused by a single bar or end ring segment faults.

- A few broken bars would produce notable torque and speed pulsations. Also, a $2Sf_1$ frequency component will show up, as expected, in the stator phase currents.

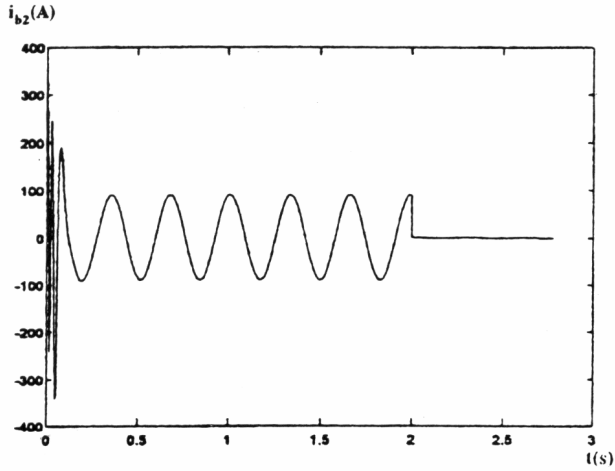


Mechanical angular velocity ω_m . Bar 2 is broken



Electromagnetic torque T_e . Bar 2 is broken

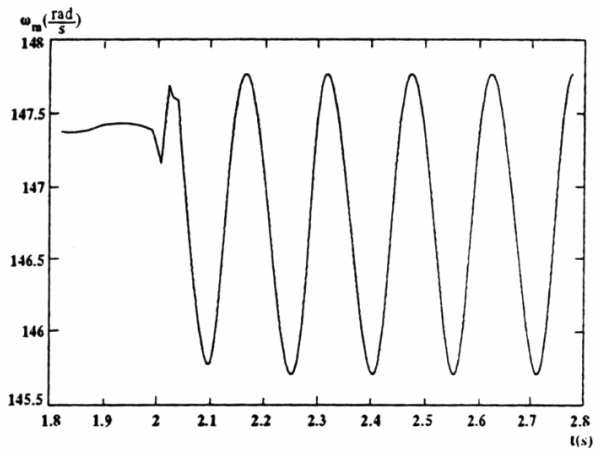
Figure 13.40 Bar 2 broken at steady-state ($T_L = 3.5\text{Nm}$)
a.) speed, b.) torque, c.) broken bar current (continued)



Current of broken bar. Bar 2 is broken

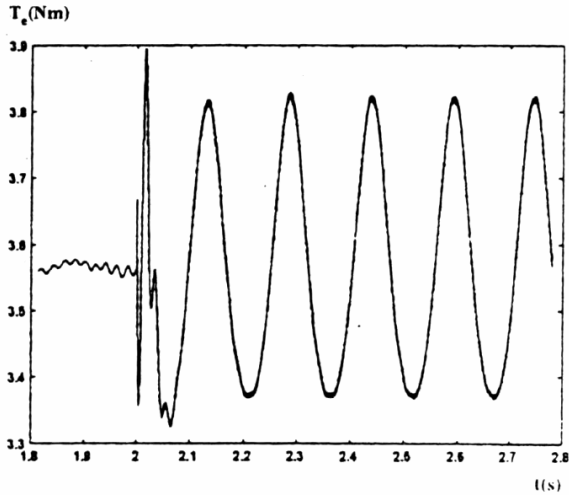
Figure 13.40 (continued)

The assumption that the interbar resistance is infinitely large (no interbar currents or insulated bars) is hardly true in reality. In presence of interbar currents, the effect of 1 to 2 broken bars tends to be further diminished. [34]



Mechanical angular velocity. Ring-section 3 is broken

Figure 13.41 End ring 3 broken at steady-state
a.) speed, b.) torque (continued)



Electromagnetic torque. Ring-section 3, is broken

Figure 13.41 (continued)

The transients of IM have also been approached by FEM, specifically by time stepping coupled FEM circuit (or state space) models. [35, 36] The computation time is still large, but the results are getting better and better as new contributions are made. The main advantage of this method is the possibility to account for most detailed topological aspects and non-linear material properties. Moreover, coupling the electromagnetic to thermal and mechanical modeling seems feasible in the near future.

13.17. SUMMARY

- Induction machines undergo transients when the amplitude and frequency of electric variables and (or) the speed vary in time.
- Direct starting, after turn-off, sudden mechanical loading, sudden short-circuit, reconnection after a short supply fault, behavior during short intervals of supply voltage reduction, performance when PWM converter-fed, are all typical examples of IM transients.
- The investigation of transients may be approached directly by circuit models or by coupled FEM circuit models of different degrees of complexity.
- The phase coordinate model is the “natural” circuit model, but its stator-rotor mutual inductances depend on rotor position. The order of the model is 8 for a single three-phase winding in the rotor. The symmetrical rotor

cage may be replaced by one, two, n, three-phase windings to cater for skin (frequency) effects.

- Solving such a model for transients requires a notable computation effort on a contemporary PC. Dedicated numerical software (such as Matlab) has quite a few numerical methods to solve systems of nonlinear equations.
- To eliminate the parameter (inductance) dependence on rotor position, the space phasor (complex variable) or d-q model is used. The order of the system (in d-q, real, variables) is not reduced, but its solution is much easier to obtain through numerical methods.
- In complex variables, with zero homopolar components, only two electrical and one mechanical equations remain for a single cage rotor.
- With stator and rotor flux space phasors $\bar{\Psi}_s$ and $\bar{\Psi}_r$ as variables and constant speed, the machine model exhibits only two complex eigen values. Their expressions depend on the speed of the reference system.
- As expected, below breakdown torque speed, the real part of the complex eigen values tends to be positive suggesting unstable operation.
- The model has two transient electrical time constants τ_s' , τ_r' , one of the stator and one of the rotor. So, for voltage supply, the stator and rotor flux transients are similar. Not so for current supply (only the rotor equation is used) when the rotor flux transients are slow (marked by the rotor time constant $\tau_r = L_r/R_r$).
- Including magnetic saturation in the complex variable model is easy for main flux path if the reference system is oriented to main flux space phasor $\bar{\Psi}_m = \Psi_m$, as $\Psi_m = L_m(i_m)i_m$.
- The leakage saturation may be accounted for separately by considering the leakage inductance dependence on the respective (stator or rotor) current.
- The standard equivalent circuit may be ammended to reflect the leakage and main inductances, L_{sl} , L_{rl} , L_m , dependence on respective currents i_s , i_r , i_m . However, as both the stator and rotor cores experience a.c. fields, the transient values of these inductances should be used to get better results.
- For large stator (and rotor) currents such as in high performance drives, the main and leakage flux contribute to the saturation of teeth tops. In such cases, the airgap inductance L_g is separated as constant and one stator and one rotor total core inductance L_{si} , L_{ri} , variable with respective currents, are responsible for the magnetic saturation influence in the machine.
- Further on, the core loss may be added to the complex variable model by orthogonal short-circuited windings in the stator (and rotor). The ones in the rotor may be alternatively used as a second rotor cage.
- Core losses influence slightly the efficiency torque and power factor during steady-state. Only in the first few miliseconds the core loss “leakage” time constant influences the IM transients behavior. The slight detuning in field orientation controlled drives, due to core loss, is to be considered mainly when precise (sub 1% error) torque control is required.

- Reduced order models of IMs are used in the study of power system steady-state or transients, as the number of motors is large.
- Neglecting the stator transients (stator leakage time constant τ_s') is the obvious choice in obtaining a third order model (with $\bar{\Psi}_r$ and ω_r as state variables). It has to be used only in synchronous coordinates (where steady-state means dc). However, as the supply frequency torque oscillations are eliminated, such a simplification is not to be used in calculating starting transients.
- The supply frequency oscillations in torque during starting are such that the average transient torque is close to the steady-state torque. This is why calculating the starting time of a motor via the steady-state circuit produces reasonable results.
- Considering leakage saturation in investigating starting transients is paramount in calculating correctly the peak torque and current values.
- Large induction machines are coupled to plants through a kind of elastic coupling which, together with the inertia of motor and load, may lead to torsional frequencies which are equal to those of transient torque components. Large torsional torques occur at the load shaft in such cases. Avoiding such situations is the practical choice.
- The sudden short-circuit at the terminals of large power IMs represents an important liability for not-so-strong local power grids. The torque peaks are not, however, severely larger than those occurring during starting. The sudden short-circuit test may be used to determine the IM parameters in real saturation and frequency effects conditions.
- More severe transients than direct starting, reconnection on residual voltage, or sudden short-circuit have been found. The most severe case so far occurs when the primary of transformer feeding an IM is turned off for a short interval (tens of milliseconds), very soon after direct starting. The secondary of the transformer (now with primary on no-load) introduces a large stator time constant which keeps alive the d.c. decaying stator current components. This way, very large torque peaks occur. They vary from 26 – 40 p.u. in a 7.5 HP to 12 to 14 p.u. in a 500 HP machine.
- To treat unsymmetrical stator voltages (connections) so typical with PWM converter fed drives, the abc-dq model seems the right choice. Fault conditions may be treated rather easily as the line voltages are the inputs to the system (stator coordinates).
- For steady-state stability in power systems, first order models are recommended. The obvious choice of neglecting both stator and rotor electrical transients in synchronous coordinates is not necessarily the best one. For low power IMs, the first order system thus obtained produces acceptably good results in predicting the mechanical (speed) transients. The subtransient voltage (rotor flux) E' and its angle δ represent the fast variables.
- For large power motors, a modified first order model is better. This new model reflects the subtransient voltage E' (rotor flux, in fact) transients,

with speed ω_r and angle δ as the fast transients calculated from algebraic equations.

- In industry, a number of IMs of various power levels are connected to a local power grid through a power bus that contains a series reactance and a parallel capacitor (to increase power factor).

Connection and disconnection transients are very important in designing the local power grid. Complete d–q models, with saturation included, proved to be necessary to simulate residual voltage (turn-off) of a few IMs when some of them act as motors and some as generators until the mechanical and magnetic energy in the system die down rather abruptly. The parallel capacitor delays the residual voltage attenuation, as expected.

- Series capacitors are used in some power grids to reduce the voltage sags during large motor starting. In such cases, subsynchronous resonance (SSR) conditions might occur.
- In such a situation, high current and torque peaks occur at certain speed. The maximum value of power bus plus stator resistance R_e for which SSR occurs, shows how to avoid SSR. A better solution (in terms of losses) to avoid SSR is to put a resistance in parallel with the series capacitor. In general the critical resistance R_{ad} (in parallel) for which SSR might occur increases with the series capacitance and so does the SSR frequency.
- Rotor bar and end ring segment faults occur frequently. To investigate them, a detailed modeling of the rotor cage loops is required. Detailed circuit models to this end are available. In general, such fault introduces mild torque and speed pulsations and $(1-2S)f_1$ pulsations in the stator current. Information like this may be instrumental in IM diagnosis and monitoring. Interbar currents tend to attenuate the occurring asymmetry.
- Finite element coupled circuit models have been developed in the last 10 years to deal with the IM transients. Still when skin and saturation effects, skewing and the rotor motion, and the IM structural details are all considered, the computation time is still prohibitive. World-wide aggressive R & D FEM efforts should render such complete (3D) models feasible in the near future (at least for prototype design refinements with thermal and mechanical models linked together).

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